Designing Electricity Auctions

Natalia Fabra
Universidad Carlos III and UCEI

Nils-Henrik von der Fehr
University of Oslo

David Harbord
Market Analysis

UC Energy Institute, August 30, 2003

UK Electricity Prices: Market Rules or Market Structure?

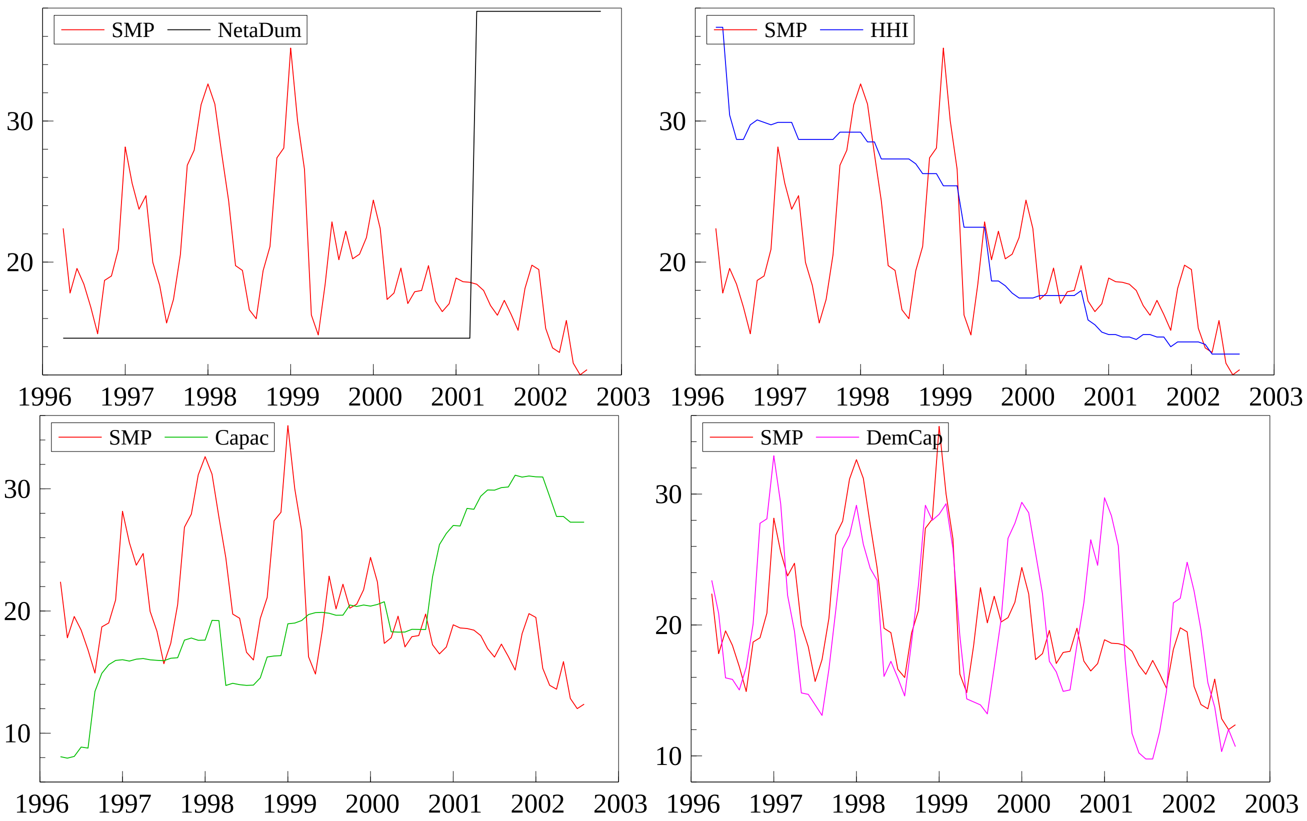

Figure 1: The SMP, NETA, HHI, Capacity and Demand

The Issues

- **Designing electricity markets:**
  - Auction format: determination of prices
  - Bid formats: number of admissible steps
  - Price-elastic demand
  - Duration of bids: short-lived vs. long-lived
  - Market Structure

- **Modelling electricity markets**

- **Aim:**
  Construct a tractable model that captures essential features of electricity markets

Structure of the Presentation

- The basic model: uniform and discriminatory

- Equilibrium analysis

- Variations on the basic model
  - Multiple Bids
  - Price-elastic demand
  - Oligopoly
  - Uncertain demand

- Conclusions

The Basic Model

Two independent **suppliers** $i = 1, 2$, with

– Productive capacities $k_i > 0$.

– Constant unit costs $c_i \geq 0$, with $c_i = 0 \leq c_2 = c$.

Demand $\theta \in (0, k_1 + k_2)$ is completely price inelastic.

Timing:

–Having observed demand, suppliers simultaneously submit price offers $b_i \leq P$ for their entire capacities.

–Outputs are determined based on the ranking of offer prices:

$$q_i(\theta; \mathbf{b}) = \begin{cases} \min\{\theta, k_i\} & \text{if} \quad b_i < b_j \\ \rho_i \min\{\theta, k_i\} + [1 - \rho_i] \max\left\{0, \theta - k_j\right\} & \text{if} \quad b_i = b_j \\ \max\left\{0, \theta - k_j\right\} & \text{if} \quad b_i > b_j \end{cases}$$

where $\rho_1 = 1, \rho_2 = 0$.

Payments

- **Uniform auction:**
  All suppliers are paid the highest accepted bid (system marginal price):

$$\pi_i^u(\theta;\mathbf{b}) = \begin{cases} \left[b_j - c_i\right] q_i(\theta;\mathbf{b}) & \text{if } b_i \leq b_j \text{ and } \theta > k_i \\ \left[b_i - c_i\right] q_i(\theta;\mathbf{b}) & \text{otherwise} \end{cases}$$

- **Discriminatory auction:**
  Suppliers are paid their own bid:

$$\pi_i^d(\theta;\mathbf{b}) = \left[b_i - c_i\right] q_i(\theta;\mathbf{b})$$

Equilibrium Analysis

Lemma 1 *In any pure-strategy equilibrium, the highest accepted price offer equals either c or P.*

Proposition 1 *There exists $\widehat{\theta} = \widehat{\theta}\,(c, k_1, k_2, P)$ such that:*

(i) (Low demand) if $\theta \leq \widehat{\theta}$, in the unique pure-strategy equilibrium the highest accepted price offer equals c.

(ii) (High demand) if $\theta > \widehat{\theta}$, all suppliers are paid prices that exceed c. A pure-strategy equilibrium exists in the uniform auction, with the highest accepted offer price equal to P, but not in the discriminatory auction.

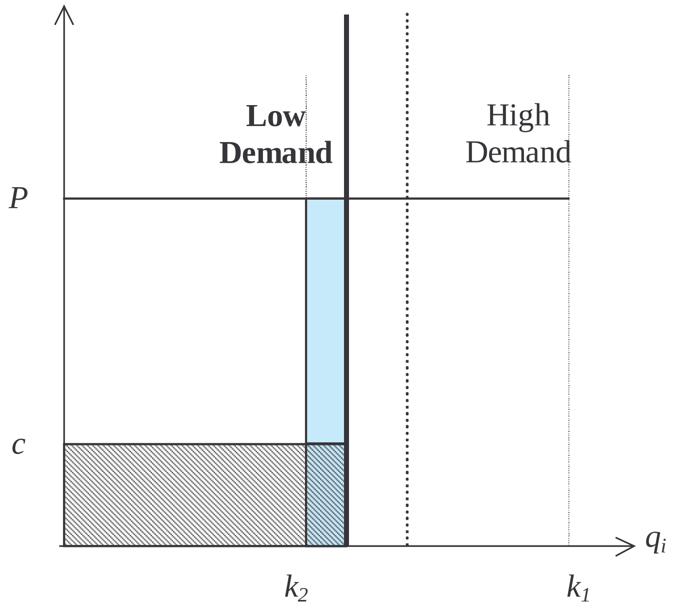

Figure 2:

The incentive of the residual supplier: Low and High demand

Sketch of the Proof

- Necessary and sufficient condition for an equilibrium with highest offer at c :

$$[c - c_i] \min \left\{ \theta - k_j, k_i \right\} - [P - c_i] \max \left\{ \theta - k_j, 0 \right\} \geq 0$$

  This expression is non-increasing in θ.
  There exists a unique $\widehat{\theta}_i$ such that the condition is satisfied iff $\theta \leq \widehat{\theta}_i$.
  Existence of the equilibrium then requires $\theta \leq \min \widehat{\theta}_i = \widehat{\theta}$.

- Necessary and sufficient condition for an equilibrium with highest offer at P :

$$[P - c_i] \max \left\{ \theta - k_j, 0 \right\} - [c - c_i] \min \left\{ \theta - k_j, k_i \right\} \geq 0$$

  Existence of the equilibrium then requires $\theta \geq \min \widehat{\theta}_i = \widehat{\theta}$.

Comparison across Auctions: A Tale of Two States

- **Low demand** $[\theta \leq \widehat{\theta}]$

  Bidding: competitive bidding with highest accepted offer c.
  Revenues: $R^u = R^d$.
  Cost efficiency: $C^u = C^d$.

- **High demand** $[\theta > \widehat{\theta}]$

  Bidding:
  Uniform: $b_1 < b_2 = P$ and/or $b_2 < b_1 = P$;
  Discriminatory: mixed strategy equilibrium, with $b_i \in (c, P]$.
  Revenues: $R^u > R^d$
  Cost efficiency: $C^u < C^d$ if in the uniform auction the equilibrium with $b_2 < b_1$ is played, $C^u > C^d$, otherwise.

Comparison across Auctions: A Tale of Two States (cont.)

- The relative incidence of low and high demand states determines the extent to which...
  - the industry is more or less competitive;
  - market outcomes differ across auctions

- **Low demand state more likely under...**
  - Capacity symmetry
  - Larger installed capacity
  - Cost asymmetry
  - Stricter regulation (low P)

Example: Increasing Installed Capacity

- Assumptions: symmetric suppliers, uniform distribution

K	1	1.2	1.4	1.6	1.8	2
ER^d	0.250	0.160	0.090	0.040	0.010	0
ER^u	0.375	0.320	0.255	0.180	0.095	0
$\frac{ER^d}{ER^u}$	0.667	0.500	0.353	0.222	0.105	na

Example: Increasing Capacity Asymmetries

- Assumptions: fixed $K = 1$, uniform distribution

k_1	0.5	0.6	0.7	0.8	0.9	1
k_2	0.5	0.4	0.3	0.2	0.1	0
ER^d	0.250	0.300	0.350	0.400	0.450	0.5
ER^u	0.375	0.420	0.455	0.480	0.495	0.5
$\frac{ER^d}{ER^u}$	0.667	0.714	0.769	0.833	0.909	1

Variations: **Multiple Unit Suppliers**

Suppliers submit (upward sloping) **step offer-price functions**: (b_{in}, k_{in}), $n \leq N_i < \infty$.

- *Equilibrium outcomes* - not equilibrium *pricing* - are essentially independent of the number of admissible steps.

Unique zero-profit equilibrium outcome in uniform auction, in contrast to continuous supply function models

- **Discrete versus Continuous bidding:**
  – Discrete bidding performs better
  – Reducing the number of steps does not affect the outcomes, but makes bidding simpler

Variations: **Price-Elastic Demand**

- Demand function: $D(p,\theta)$, with standard assumptions
  – The parameter θ defines a family of demand functions s.t. if $\theta_1 < \theta_2$, $D(p,\theta_1) < D(p,\theta_2)$.
  – Residual monopoly price: $p_i^r(\theta) = \arg\max_p \left\{ p \min\left[D(p,\theta) - k_j, k_i \right] \right\}$.
  – Effective residual monopoly price: $P_i^r = \min\left\{ p_i^r, P \right\}$.

- **Equilibrium Analysis**: extension of Proposition 1
  – There exists a unique threshold $\widehat{\theta}$ such that equilibrium outcomes are of the low-demand case iff $\theta \leq \widehat{\theta}$, and of the high-demand case otherwise.

Price-Elastic Demand (cont.)

- The **comparison across auction formats** is similar:
  Plus, allocative efficiency gain in the discriminatory auction.

- **Demand elasticity improves market performance:**
  – Reduces equilibrium price
  – Makes the low-demand state more likely, i.e. larger $\widehat{\theta}$.

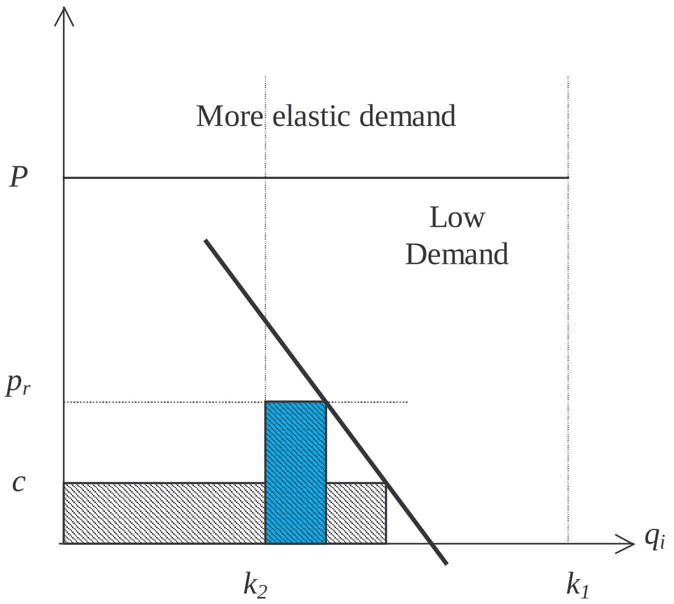

Figure 3: The effects of increasing demand elasticity

Example: Increasing Demand Elasticity

- Assumptions: symmetric suppliers, uniform distribution

b	0	0.025	0.050	0.075	0.100	0.125	0.150
ER^d	0.250	0.226	0.203	0.183	0.163	0.146	0.130
ER^u	0.375	0.350	0.327	0.304	0.282	0.260	0.240
$\frac{ER^d}{ER^u}$	0.667	0.646	0.621	0.602	0.578	0.562	0.542

Variations: **Oligopoly**

N suppliers with $k_1,\ldots,k_N$ and $c_1 = 0 \le c_2 \le \ldots \le c_N = c$.

Proposition 2 *There exists* $\widehat{\theta}^-$ *and* $\widehat{\theta}^+$, $\widehat{\theta}^- \le \widehat{\theta}^+$, *such that*
(i) (low demand) if $\theta \le \widehat{\theta}^-$, *in any equilibrium the highest accepted price offer is at or below* c*;*
(ii) (high demand) if $\theta > \widehat{\theta}^+$, *in any equilibrium suppliers are paid prices that exceed* c*;*
(iii) $\widehat{\theta}^- = \widehat{\theta}^+ = \widehat{\theta}$ *if* $k_N \ge \max_{j<N} k_n$.

– Low-demand: competitive, but not necessarily efficient
– Coexistence of competitive and non-competitive equilibria

Variations: **Symmetric Oligopoly**

Low-demand state (i.e., highest accepted price offer no higher than c) iff $\theta \leq \frac{N-1}{N}K$, high-demand state otherwise

- **De-concentrating market structure:**
  – Reduces incidence of high-demand state.
  – In the discriminatory auction, intensifies price competition in high-demand state.

Example: Increasing the Number of Suppliers

- Assumptions: symmetric suppliers, uniform distribution

N	2	3	4	5	10	100	∞
ER^d	0.250	0.167	0.125	0.100	0.050	0.005	0
ER^u	0.375	0.278	0.219	0.180	0.095	0.010	0
$\frac{ER^d}{ER^u}$	0.667	0.600	0.571	0.556	0.526	0.503	0.5

Variations: **Uncertain Demand**

Demand θ takes values in $\left[\underline{\theta}, \overline{\theta}\right] \subseteq (0, k_1 + k_2)$ according to $G(\theta)$
Similar results as above if $\overline{\theta} < \widehat{\theta}$ (low) or $\underline{\theta} > \widehat{\theta}$ (high)

Lemma 2 *Assume $\underline{\theta} < \widehat{\theta} < \overline{\theta}$. There does not exist an equilibrium in pure strategies in either auction. In the unique mixed-strategy equilibrium suppliers submit bids that strictly exceed c.*

- The two auction formats are equivalent if suppliers are symmetric; the comparison is unclear otherwise.

- With symmetric suppliers, long-lived bids perform better.

Variations: **Vickrey Auction**

Payments: Every supplier is paid the opportunity cost of its output; i.e. the rival's rejected offer times its excess capacity plus P for any remaining output.

$$\pi_i^v(\theta;\mathbf{b}) =$$
$$\begin{cases} \left[b_j - c_i\right] q_i(\theta,\mathbf{b}) & \text{if } b_i \leq b_j;\ \theta \leq k_j \\ \left[b_j - c_i\right]\left[k_j - q_j(\theta,\mathbf{b})\right] + [P - c_i]\left[\theta - k_j\right] & \text{if } b_i \leq b_j;\ \theta > k_j \\ [P - c_i]\, q_i(\theta;\mathbf{b}) & \text{if } b_i > b_j. \end{cases}$$

- **Equilibrium Bidding:**
  For any realization of demand, there exists a unique equilibrium in weakly dominant strategies in which suppliers offer prices at marginal cost.

Vickrey Auction (cont.)

- **Comparison with uniform and discriminatory:**

  The Vickrey auction always results in **cost efficiency**.

  But can result in **large payments**, and thus be outperformed by the uniform or discriminatory auctions.

Conclusions

- **Equilibrium outcomes:**
  – Competitive with low-demand, non-competitive otherwise
  – Incidence of low-demand state depends on market structure, technology, demand elasticity and price caps, but not on the auction format.

- **Comparison across auction formats:**
  – Payments: discriminatory outperforms uniform.
  – Efficiency: depends on equilibrium played in uniform.
  – Regulatory measures: more effective with discriminatory

Conclusions (cont.)

- **Market structure versus market design:**
  Switching to discriminatory may reduce prices as much as:
  – doubling the number of players;
  – increasing the capacity of two symmetric duopolists by nearly 40%.

- **Demand Elasticity:**
  – Increasing demand elasticity not only reduces prices in high demand state, it also reduces incidence of high demand states
  – Switching to a discriminatory may lead to a similar reduction in prices as increasing demand elasticity from 0 to 0.15.