Designing and Structuring Auctions for Firm and Interruptible Gas Supply Contracts in Colombia

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Tasks 1 and 2 Report

08 May 2011

Abstract

The CREG has commissioned Market Analysis to specify the conceptual design and the structure of natural gas auctions to be implemented in Colombia, as part of a Colombian "gas release" program. This project builds on previous work for the CREG by Cramton (2008) and Harbord (2010). The required tasks include the specification of: (i) the products or contracts to be sold in the auctions (Task 1); (ii) the overall conceptual design of the auction (Task 2); and (iii) the detailed auction rules (Task 3). This report contains the results of Tasks 1 and 2 of the project. A subsequent (Task 3) report will specify the rules for the auction design to be implemented, following consultations with the CREG and the industry.

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1 Introduction

The CREG has commissioned Market Analysis to specify the conceptual design and the structure of natural gas auctions to be implemented in Colombia, as part of a Colombian "gas release" program. This project builds on previous work for the CREG by Cramton (2008) and Harbord (2010). The required tasks include the specification of: (i) the products or contracts to be sold in the auctions (Task 1); (ii) the overall conceptual design of the auction (Task 2); and (iii) the detailed auction rules (Task 3). The latter includes the auction bid functions, i.e. the demand and supply curves to be submitted by buyers and sellers in the auctions, the auction activity rules (if any), reserve prices (if any), the auction pricing and allocation mechanism, and any special rules required to address potential market power issues.

This report contains the results of Tasks 1 and 2 of the project and is organized as follows:

- Section 2 provides an overview of the Colombian gas supply market as it operates today.
- Section 3 considers the products to be sold in the auctions, and the related issue of auction frequency.
- Section 4 describes and compares two basic auction designs: the simultaneous, ascending clock auction and the simultaneous sealed-bid ("assignment" or "product-mix") auction.
- Section 5 discusses some other important issues, such as whether or not the auction products should be location (or field) specific, and the possible sale (or resale) of firm conditional contracts in the auctions by producers and gas-fired power plants.
- Section 6 summarizes our initial recommendations and conclusions on these topics where we have any.

A subsequent Task 3 report will specify the rules for the auction design to be implemented, following consultations with the CREG and the industry.

2 Overview of the Colombian Upstream Gas Market

This section provides a brief overview of the upstream gas supply market in Colombia.[1]

2.1 Supply

All natural gas consumed in Colombia is domestically produced with roughly 90% coming from two main fields: Guajira on the Caribbean coast and the Cusiana fields in the interior. Several minor fields account for the remaining 10%.

[1] All information for this section was provided by the CREG.

Guajira has about 41% of Colombia's reserves (but this is declining over time), and currently provides 65% of production. The field is jointly operated by Ecopetrol, the state-owned oil company, and Chevron Texaco. In 2009, average production of the Guajira fields was approximately 695 GBTU per day. Gas from these fields is delivered to the entry point of Ballena, and is shipped to the inland part of the country, the Atlantic/Caribbean coast, and to Venezuela.

Cusiana has about 49% of total Colombian gas reserves and provides approximately 21.7% of current supply, producing approximately 226.4 GBTU per day. Until recently, the fields were operated jointly by Ecopetrol, BP, and Tepma/Total. In January 2011 Equion Energia Ltd, a joint venture between Ecopetrol and Talisman Energy, acquired all of BPs oil and gas production assets in Colombia. Ecopetrol owns 51% of the new company and Talisman the remaining 49%.

Other minor fields produce around 105.7 GBTU per day: La Creciente, 42.8 GBTU; Payoa, 19 GBTU; other, 43.8 GBTU. There is also a new field in Gibraltar, expected to produce 30 GBTU per day by the end of 2010.

The upstream gas market in Colombia is highly concentrated. Table 2.1 shows average daily production by company in 2009 and 2010, and Table 2.2 shows average daily gas production by field and company. The 2009 Herfindahl-Hirschman Index (HHI) for gas supply was 4529, and the degree of concentration is increasing as Ecopetrol acquires further control of the Cusiana fields production.

Table 2.1. Gas supply by company in 2009/10

Company	GBTUD	Share	GBTUD	Share
	2009		2010[2]	
Ecopetrol	603	61%	624	63%
Chevron	228	23%	233	23%
BP[3]	59	6%	54	5%
Tepma/Total	34	4%	32	3%
Others	61	6%	76	8%
TOTAL	985	100%	1018	100%

[2] January-October 2010.
[3] Now Ecopetrol/Talisman.

Table 2.2 Gas supply by company and field in 2009/10

Field	**Company**	**GBTUD**	**Share**
La Guajira	Ecopetrol	435	66%
	Chevron	228	34%
Cusiana	Ecopetrol	136	60%
	BP[4]	59	25%
	Total	34	15%
La Creciente	Pacific Rubiales	43	100%
Smaller fields	Ecopetrol	32	63%
	Others	18	37%

2.1.1 Production Sharing and Association Contracts

The Guajira fields are jointly operated by Ecopetrol and Chevron Texaco under an association contract which was extended in 2003 to continue for the economic life of the field. Ecopetrol's share of production under this contract is 65.6%, and Chevron's is 34.4%.

The Cusiana fields are operated by Ecopetrol, Equion, and Tepma/Total under association contracts. Ecopetrol's share is approximately 60%, Equion's 24.8% and Tepma's 15.2%.

Under association contracts, production and investment decisions are not made independently by the individual companies. Hence the producers are not really independent competitors in supply. Thus the concentration measures described above likely underestimate the degree of market power held by the large producers in upstream gas supply.[5]

2.2 Demand

Demand for gas in Colombia falls into four main categories: residential and commercial (19%); industrial (45%); electricity generation (24%); and vehicles (11%). Demand is located on the Atlantic/Caribbean coast (34%) and in the interior (52%). Exports to Venezuela currently account for 14% (approx. 150 GBTUDs) of demand.

[4] Now Ecopetrol/Talisman.

[5] Based on results from Stiglitz (1976), Pindyck (1985) suggested that standard concentration measures may significantly overestimate the degree of market, or monopoly, power held by producers of exhaustible resources, such as gas or oil. In one polar case (isoelastic demand and zero extraction costs), Stiglitz (1976) found that a resource monopolist will produce exactly the same output over time as a perfectly competitive industry, so a monopolist will have no market power at all. Subseqently, Pindyck (1987) showed that these conclusions do not withstand more realistic modelling and concluded that, "*in general, resource depletion will reduce monopoly power and increase the (initial) monopoly production rate, bringing it closer to the competitive rate. However, for most resources, ... depletion does not significantly limit potential monopoly power. ... One cannot use resource depletion as an argument for giving extractive resource industries "special treatment" vis-a-vis the antitrust laws.*" Gaudet and Lasserre (1988) reach similar conclusions.

Table 2.3 Contract positions of purchasers, 2009-2010 (MBTUDs)

	Demand 2009	Market share	Demand 2010	Market share
PDVSA	150,000	16.17%	150,000	16.60%
E2	140,000	15.09%	134,583	14.89%
GECELCA	100,700	10.86%	57,260	6.34%
GAS NATURAL	95,192	10.26%	96,715	10.70%
EPM	83,787	9.03%	84,553	9.36%
ISAGEN	59,000	6.36%	59,000	6.53%
ECOPETROL REFINERIA	55,377	5.97%	74,019	8.19%
TERMOFLORES I, II, III	52,021	5.61%	52,021	5.76%
EPSA	36,000	3.88%	36,000	3.98%
MERIELECTRICA	32,800	3.54%	32,800	3.63%
ABONOS COLOMBIANOS	20,500	2.21%	20,500	2.27%
CERROMATOSO	16,000	1.73%	16,000	1.77%
TERMOEMCALI	16,000	1.73%	16,000	1.77%
REFICAR	14,940	1.61%	14,940	1.65%
CHEC	9,624	1.04%	9,624	1.06%
ECOPETROL	9,102	0.98%	6,254	0.69%
DINAGAS	6,563	0.71%	7,768	0.86%
ALCANOS DE COLOMBIA	8,100	0.87%	13,696	1.52%
OTHERS	21,825	2.35%	22,016	2.44%
TOTALS	**927,530.67**	**100%**	**903,748.50**	**100%**

Approximately 49% of demand on the north coast comes from thermal electricity generators. The interior also has significant gas-fired generation capacity, but these units generate little or no electricity in a typical year, since hydro resources are less expensive when there are sufficient water resources.

The main consumption points are located in the major urban centers (e.g. Bogotá, Cali, Barranquilla, and Medellín among others), and where gas-fired power plants and refineries are located. These plants are located in the southern part of the country, near to Barranquilla, and in the central interior region near to Barrancabermeja.

The market is relatively unconcentrated on the demand side with approximately 37 companies in the market (including exports). Table 2.3 below shows the annual average contract positions of the larger consumers and shippers from 2009 to 2010. Color coding indicates companies under common ownership.

The largest single purchaser of gas in 2010 was PDVSA (Petróleos de Venezuela) for export, followed by E2 (Energia Eficiente), a gas trader located on the Atlantic Coast, followed by the gas distribution companies Gas Natural (Bogota) and EPM (Empresas Publicas de Medellin).

Table 2.4 shows the purchasers of gas contracts by field from 2008-2012. Twenty-seven (27) companies purchase gas from the Guajira field (15 independent buyers once common ownership is accounted for), twenty-seven (27) from the Cusiana fields (24 once common ownership is

Table 2.4 Gas purchasers by field

GUAJIRA	CUSIANA	LA CRECIENTE
TERMOFLORES II	GAS NATURAL	PROELECTRICA
TERMOFLORES III	EPM	GECELCA
EPM	E2	MANUFACTURAS SILICEAS
GECELCA	ISAGEN	FAVIDRIO
ISAGEN	TERMOEMCALI	PRODS FAMILIA SANCELA
CHEC	TERMOCOA	TOPTEX
EPSA	ECOPETROL	SIDENAL
MERIELECTRICA	PETROBRAS	GNV
CERROMATOSO	CEMEX	EPM
CEMENTOS ARGOS	MANSAROVAR	PETROMIL
ABONOS COLOMBIANOS	GASES DEL LLANO	
DRUMMOND	GASES DEL CUSIANA	
GASES DEL CARIBE	DINAGAS	
SURTIGAS	ENERCA	
GASES DE LA GUAJIRA	MADIGAS	
GAS NATURAL DEL CENTRO	ESTACIÓN BOMBEO	
EDALGAS	ALCANOS DE COLOMBIA	
GASES DE OCCIDENTE	COLINVERSIONES	
ALCANOS DE COLOMBIA	EFIGAS	
GAS NATURAL	GASES DE OCCIDENTE	
GAS DEL RISARALDA	GAZEL	
GASES DEL QUINDIO	PLEXA	
GAS NATURAL DEL CESAR	TERMOYOPAL	
E2	PERENCO	
PDVSA	GECELCA S.A. E.S.P.	
REFICAR	METROGAS S.A. E.S.P	
ECOPETROL REFINERIA	TRNS DE GAS INTERNACIONAL	

accounted for), and ten (10) from La Creciente. Companies shaded in green do not hold contracts currently but have either purchased gas contracts in the past or have participated in recent auctions, and hence are potential or actual market participants.

2.3 Primary Market Contracts

The vast majority of gas in the primary market is sold by producers under either firm or interruptible contracts with durations varying from one year (approx. 40%) to nine years (for some gas-fired power plants). The majority are take-or-pay contracts with the minimum percentage of "take" varying from 25% to 70% for the gas-fired power plants, and with 100% levels of “take” not being unusual.[6]

Gas supply contracts from the Guajira field are sold at a regulated price, currently \$4.25 per MBTU, indexed twice annually to US Gulf fuel oil prices.[7] The prices of gas supplied from other fields are unregulated.

In December 2009, Ecopetrol held auctions for 32,821 MBTUDs in five-year firm gas contracts

[6] See Milgrom *et al* (2011) "Report on Existing Natural Gas Contracts in Colombia" for further details on contracting practices in Colombia.

[7] Prior to December 2010, Guajiria prices were indexed twice a year to New York fuel oil prices.

with take-or-pay levels of 100% from the Cusiana field, resulting in a price of \$6.14/MBTU. BP/Tempa held auctions in 2010 for 40,600 MBTUDs in five-year firm gas contracts with 100% take-or-pay levels at a clearing price of \$4.73/MBTU.

Firm transport contracts are sold at regulated prices, except for non-regulated users and marketers selling gas in the non-regulated market that have agreed on other prices with the transporter. The form and duration of these contracts are freely negotiated between shippers and TSOs.

3 Auction Products and Frequency

In most real-world auction problems, the first questions that must be answered concern product definition. What is being sold (or bought)? What should be included in each lot? How is the commodity to be defined or the terms of the contract set? The answers to these questions can have important implications for market rules and performance. An important principle of auction design is simplicity: auctions work better, and are likely to be more liquid, with standardized as opposed to differentiated products, and when the number of products is kept to a minimum.

A related and important issue in the context of Colombian gas release auctions is auction frequency. More frequent auctions may, for instance, allow us to reduce the number of products to be sold in each auction (the number of different contract durations, for example), but may also complicate buyers' calculations if they view different auctions as "substitutes", and increase the risk of strategic behavior by sellers.

We consider these issues in this section.

3.1 Auction Products

A variety of options exist for the contracts to be sold in the Colombian gas auctions.[8] Cramton (2008) suggested annual auctions for standardized take-or-pay contracts for firm gas, with a lot size of 10 MBTUDs, contract durations from one to five years, and indexing to the Colombian Producers' Price Index (or another suitable index to avoid inflation risk). Any buyer of a product would win quantity from all producers in proportion to the quantity offered by each seller.[9]

Frontier Economics (2010, pp. 34-35) proposed one-year, 100% take-or-pay firm and interruptible contracts with the same start date (beginning three months from the date of each

[8] A related study by Auctionomics and FTI on the standardization of gas supply and transport contracts in Colombia will ultimately inform and influence the design of the products to be sold in the auctions.

[9] There are many possible ways of allocating contracts to buyers after the auction. Cramton's suggestion could result in a very small purchaser needing to sign contracts with two or more producers (and subsequently nominating gas from multiple producers each day). Another possibility would be to assign buyers to producers in a manner which minimizes the number of contracts signed by each purchaser.

annual auction), with a standard lot size of 1 MPCD.[10] Frontiers' suggested lot size is thus one hundred times larger than the lot size suggested by Cramton.

3.1.1 Lot size and indexation

The smallest purchaser of gas in Colombia in 2009 was Surtigas which demanded 22 MBTUDs, while the largest was PDVSA with a demand of 150,000 MBTUDs. So a lot size between 10 MBTUD and 20 MBTUD appears to be required if the demands of smaller purchasers are to be accommodated in the auctions.

A number of companies in Colombia have suggested that contract prices be indexed to the Colombian Producer Price Index (IPP), consistent with Cramton's (2008) proposal. In Ecopetrol's recent auctions contract prices were indexed monthly to US Gulf fuel oil prices. If liquid secondary markets are eventually established, indexation to spot market prices might be adopted as is done in most European and North American markets.

3.1.2 Contract start dates and durations

Auctions work best if the products offered are substitutes from the point of view of the buyers.[11] Market-clearing prices may not exist when goods are not substitutes, or there may be multiple, unrankable equilibria (see Milgrom 2007; Klemperer 2010; Cramton *et al.* 2007). Auction designs for complements (so-called combinatorial or "package" auctions) exist,[12] and have recently been used to sell radio spectrum in a number of European countries, but are considerably more complex and can pose difficult computational problems for bidders (see Milgrom 2007; 2011).

For this reason, there is a strong argument for having each contract sold in any particular auction begin on the same date. For example, contracts sold in an auction held on 1 January 2012 would all have start dates of say 1 July 2012, but may be of varying durations (e.g. one year, two years etc). This makes it more likely that the different contracts will be viewed as substitutes by the buyers.

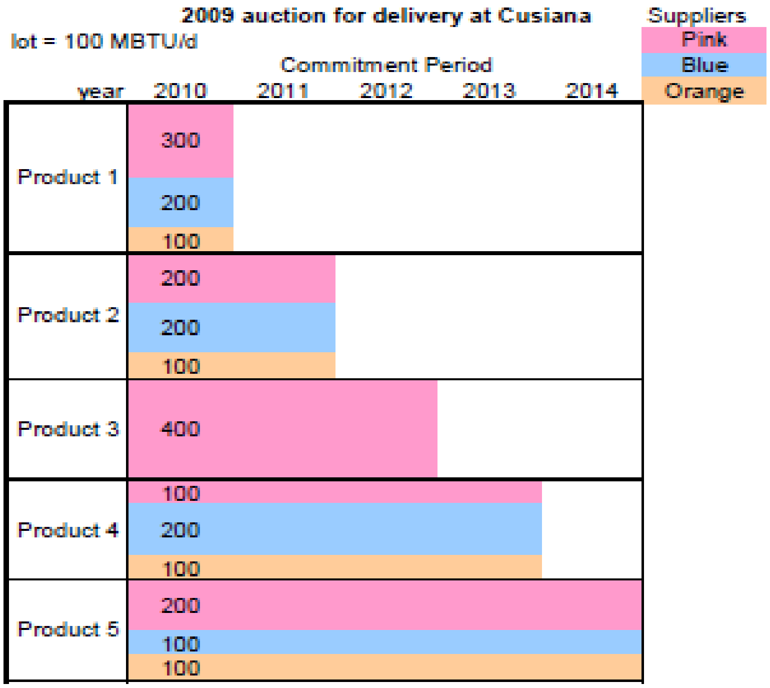
Figure 2 in Cramton (2008) illustrates this. It shows an example of products for the 2009 auction for delivery at Cusiana. There are three suppliers, Pink, Blue, and Orange, each offering quantity for up to five products. All products start in 2010 with durations from one to five years. For the one-year product, Pink is offering 300, Blue 200, and Orange 100. Any buyer of a product

[10] An MBTUD is one million BTUs (British Thermal Units) per day, and an MPCD is one million cubic feet per day, which is roughly equivalent to 1,000 MBTUDs (or 1 GBTUD).

[11] Roughly speaking, two goods are substitutes if an increase in the price of one of them results in an increase in demand for the other.

[12] Items are "complements" when a set of items has greater utility than the sum of the utilities for the individual items (for example, a pair of shoes is worth more than the value of a left shoe alone plus the value of a right shoe alone). An auction is "combinatorial" when bidders can place bids on combinations of items, called "packages," rather than on just individual items.

Figure 2. Sample supply of products



would win quantity from all producers in proportion to the quantity offered by each supplier. Thus, a buyer winning six lots of product 1 would get three lots from Pink, two lots from Blue, and one lot from Orange.

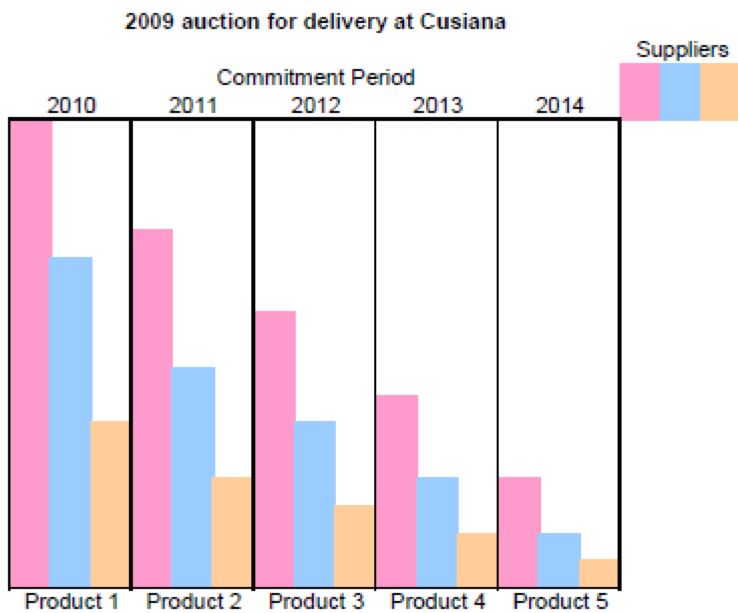
Figure 3 in Cramton (2008) shows an alternative arrangement in which the products are defined as firm gas in a particular year. With this structure the products are complements for the buyer, not substitutes, i.e. firm gas supplied in 2012 is not a substitute for gas supplied in 2010. Auctions for complements are generally more complex, and have more challenging price formation issues than auctions for substitutes, as noted above.

3.2 Auction Frequency

Cramton (2008) suggested that Colombian gas auctions be held annually, as did Frontier Economics (2010). Most of the companies we have met with in Colombia appeared to accept this suggestion.

Auctions held with higher frequency than the duration of contracts may allow for a wider set of contracts being available on the market at any given time. For example, an auction for three-year contracts could be held every year, with each set of contracts covering the subsequent three years. This would mean that at any given time a set of three different contracts would be

Figure 3. Alternative with supply by year



available in secondary markets: a contract covering the next year, a contract covering the two next years and a contract covering the next three years.

3.3 Initial Recommendations

The specification of the firm and interruptible (or conditional firm - see Section 5.3 below) contracts to be sold in the auctions will be largely determined by the results of the companion study concerning the standardization of contracts in Colombia, so we have not discussed this topic here.

Most of the companies we spoke with in our consultations considered that contract durations of one to five years would be adequate if auctions are held annually, although some argued in favour of longer-term contracts. One distribution company suggested that contracts with different start dates be sold in each auction, but most companies appeared to concur that only contracts with the same start dates should be offered. There was apparent agreement on a lot size of 10-20 MBTUDs.

In light of the discussion above and our consultations with the industry, our initial recommendations are:

- **lot size:** 10-20 MBTUDs
- **contract durations:** one-year and five-year contracts

- **contract start dates:** six months or one year from the date of the each auction
- **contract indexation:** the Colombian Producer Price Index (IPP) or other suitable index for contracts exceeding one year
- **auction frequency:** annual

These initial recommendations are likely to be revised following further consultations with the CREG and the Colombian industry.

Two other issues relating to the products to be sold in the auctions remain to be considered in more detail. These are: (i) whether the products in the auctions should distinguish gas supplies by location, i.e. specify the field from which the gas is to be delivered, or whether all gas offered in the auctions can be treated *as if* it originated in the same location; and (ii) whether gas-fired power plants or upstream producers should be able to sell gas in the primary auctions in conditional firm contracts or in some related form. These more open questions are discussed in greater detail in Section 5 below, where we also consider the issue of the simultaneous sale of gas and transport capacity.

4 Auction Design

There are two main alternative auction designs for selling multiple substitute goods, such as gas supply contracts from different fields or with different durations. These are the **simultaneous ascending clock auction**, which has already been used by producers in Colombia to sell gas supply contracts, and the recently proposed **simultaneous sealed-bid auction,** also known as "assignment" or "product-mix" auctions. We describe these two auction designs in some detail in this section, and consider the main advantages and disadvantages of each in light of the specific characteristics of the Colombian gas market.

4.1 The Simultaneous Ascending Clock Auction

The simultaneous ascending auction was introduced in 1994 to sell spectrum licenses in the United States and has subsequently been adopted, with some variations, for numerous spectrum auctions worldwide, and to sell various other goods, like electricity and gas (see Milgrom, 2004; Klemperer, 2004; and Ausubel and Cramton 2010).

In an ascending-clock auction, bidding takes place in discrete rounds. For example, in an auction for a single divisible good, such as the total quantity of a specific gas supply contract, in each round the auctioneer announces a new price for the good and each bidder submits a bid that specifies the quantity of the good on sale that it is interested in acquiring at the announced price. After each round, the bidding results are posted, typically consisting of the total excess demand at the current auction price, and possibly including additional information regarding each individual bidder's demand.[13] In subsequent rounds, the price increases as long as there is excess demand for the good on sale. The auction terminates when there is no excess demand, and each winning bidder is awarded a quantity equal to its current demand and pays the current auction price for each unit won (except that marginal bidders may be rationed).

When there are multiple goods on sale, ascending-clock auctions for each good are run simultaneously, in order to allow bidders to compare current prices for all goods and observe the likely outcome of the auctions for all goods, when deciding how much to bid on each single good. In each round the auctioneer announces one price for each good and bidders indicate their demand for each good. In subsequent rounds, the auctioneer increases the price of any good with excess demand but does not change the prices of goods with no excess demand. The auction terminates when there is no excess demand on any good.[14]

Because a bidder's demand for one good usually depends on the prices of other goods, to ensure an efficient allocation it is essential to allow bidders to "switch" their demand between the different goods during the ascending auction, as prices change. For example, when the price of one good increases compared to price of another good, a bidder may prefer to acquire less of the first good, and more of the second; hence it may want to reduce demand on the first good and increase demand on the second one. In order to achieve an efficient allocation, the auction mechanism should allow bidders to do this. Switching demand when relative prices change is natural when goods are substitute, as it is arguably the case with different types of gas supply contracts covering similar periods of time and/or delivered at different locations.

Switching demand among the various goods on sale can be allowed in a simultaneous ascending-clock auction, subject to an "activity rule" that controls a bidder's eligibility to make new bids on the various goods during the auction. The activity rule creates pressure on bidders to bid actively from the start of the auction in a way that is consistent with the bidder's true preferences (e.g., avoiding "sniping"),[15] thus increasing the pace of the auction and the amount of information available to bidders during the auction.

For example, the activity rule may require that a bidder cannot increase the total quantity demanded as prices rise. With such an activity rule, a bidder can switch demand between goods from one round to the other, so long as the total quantity it demands does not increase. An alternative, and more restrictive, activity rule requires that a bidder's demand for each good weakly decreases as prices raise. This second rule, however, prevents bidders from switching and

[13] See Harbord and Pagnozzi (2008) for a discussion of the types of information which might be communicated to participants in the Colombian firm energy auctions.

[14] For a more detailed description of the simultaneous ascending auction see, e.g., Ausubel and Cramton, 2004 and 2010.

[15] Sniping consists in a bidder only bidding at the very end of the auction for the goods it is interested in, in order to foreclose competing bids and/or to avoid revealing information about its true intention and valuations.

may produce inefficiency.

There are, however, some potential problems created by the possibility of bidders' switching demand and by the discreetness of the price increments chosen by the auctioneer in simultaneous ascending clock auctions. In the remaining of this section, we discuss these problems.

Although the possibility of switching demand is essential to ensure an efficient allocation with substitute products, it may lead the auction to terminate with excess supply. This may happen, for example, when a bidder switches demand away from a good and causes total demand on that good to be lower than supply. A possible, partial, solution to this problem is to allow the auctioneer to decrease the prices of some or all goods, until the point at which excess supply is eliminated, and ration bidders. However, to eliminate excess supply the auctioneer may need to substantially decrease prices after the end of the auction and, even if it does so, the simultaneous ascending auction does not guarantee finding the exact market-clearing prices. This is illustrated in Examples 1 and 2 in Annex A.

A related problem is that, because the price increments between rounds are discrete, in the ascending auction the price of a good may increase above its market-clearing level, thus causing excess supply. The risk of generating excess supply (and, more generally, the problems created by discrete price increments) can be partially solved by using "intraround" bids, which allows bidders to express their demands for each good at each price between the start-of-round price and the end-of-round price chosen by the auctioneer, in any round (Ausubel and Cramton, 2004).[16] But while intraround bidding is more effective for a bidder who simply wants to reduce its demand for a good, it is much less helpful for a bidder who wants to switch demand between goods (or in general when the price at which it want to reduce its demand for one of the goods also depends on the prices of other goods). The reason is that, with intraround bids, a bidder cannot condition his bid for one of the goods on the prices of other goods; hence, it cannot indicate at what *relative* prices it wants to switch demand (see Example 3 in Annex A). By contrast, a simultaneous sealed-bid auction allows bidders to do this, as we discuss in the following subsection.[17]

To eliminate some of the problems created by the possibility of bidders' switching demand, the auctioneer may prevent switching when it results in excess supply on the good from which a bidder switches demand away. However, preventing switching in these cases may result in an inefficient allocation of the products in a simultaneous ascending auction (see Example 4 in Annex A). An intermediate solution is to allow bidders to switch demand only up to the point

[16] By allowing bidders to express their preferences at more possible prices, but without observing the intraround bids of other bidders, intraround bidding makes an ascending auction more similar to a sealed-bid one.

[17] Of course, a more complex type of intraround bidding allows bidders to express more complex preferences and make bids that depend on relative prices, as described in Section 4.2. But this would essentially result in the use of a multi-stage, simultaneous sealed-bid auction.

at which demand equals supply on the good from which they are switching demand from, as suggested by Cramton (2008). This is an effective solution if the auctioneer can determine the exact relative price at which the bidder is willing to switch between two goods — i.e., the price at which it is indifferent between switching or not, so that it is willing to switch only some of its demand. But, as discussed in the previous paragraph and in Example 3 in Annex A, often an ascending auction does not manage to identify this relative price precisely, even if intraround bidding is allowed.

To determine the quantities on sale in the ascending auction, each supplier may announce the quantity of each good that it is willing to produce and the minimum price it is willing to accept before the auction. Supplier may also be allowed to announce a supply curve before the auction. The total quantities of the various goods on sale (which may depend on the auction prices) are determined by aggregating the individual sellers' demand. It is also possible to allow suppliers to make bid-offers during the auction depending on the announced price — i.e., run a "two-sided" simultaneous ascending auction.

4.1.1 Cramton's simultaneous ascending clock auction for Colombian gas contracts

Cramton (2008) proposed a simultaneous ascending clock auction design for firm gas contracts in Colombia, with the following features:

1. A mandatory auction for producers in which suppliers must sell all of their firm gas contracts in the auction (whereas a voluntary auction would allow producers to also sell long-term gas contracts bilaterally).

2. A single auction including all unregulated fields and producers, to allow buyers to see all the options for long-term gas contracts, and to arbitrage across the substitute products, enhancing price formation and reducing transaction costs.

3. Standardized contracts or products as discussed in Section 3 above.

4. Seller commitment to supply schedules before the auction starts, to prevent them from adjusting their offers in response to revealed demand during the auction. Supply schedules specify the quantity offered of each product, and reserve prices. Each seller decides before the auction how to split its quantity between contract durations, with no requirement that quantity be offered for all products, or in any particular proportion.

Cramton (2008) did not envisage the Guajira producers participating in the auction, although there appears to be no barrier to extending the auction in this way, once the Guajira price is deregulated. Nor did the Cramton proposal include provisions to control the market power of the larger producers, such as Ecopetrol. Proposals for doing so were presented in Harbord (2010).[18]

Cramton proposed a simultaneous ascending clock auction, similar to that used in the firm energy market (see Cramton and Stoft 2007; Harbord and Pagnozzi 2008). In the proposed auctions for gas contracts, however, there are more products, reflecting multiple delivery points and multiple contract durations. The motivation for using a dynamic auction, rather than a sealed-bid auction, as explained in Ausubel and Cramton (2004), is to allow for "price discovery", which Cramton (2008) argues is especially useful when there are many goods or contracts, so that bidders can freely arbitrage across the products.

The ascending clock auction is as described immediately above. The activity rule proposed in Cramton (2008) was the standard one: each bidder must bid a (weakly) downward sloping aggregate demand curve throughout the auction. This activity rule imposes no restriction on the ability of the bidder to arbitrage across the products since the restriction is with respect to the aggregate quantity demanded, not the quantity for any individual product.

As discussed above, switching or demand reductions (i.e. arbitraging across products) means that it is possible for a product to go from excess demand to excess supply. This is prevented in Cramton (2008) by rules which specify: (i) that reductions are only accepted to the point where demand equals supply. and (ii) switches are similarly restricted to prevent excess supply. With these restrictions, once a product has excess demand, it is guaranteed that the product's full quantity will be sold.

Figure 6 from Cramton (2008) illustrates how the clock auction works with many products. The top row indicates the supply offered for each of the five products. Each product has the same starting price of $5. At this price, all but one of the products has excess demand; only the 3-year product does not. As a result, for round 2, the price increases for all products, except the 3-year product. Notice that the 1-year and 5-year products increase slightly faster, since these products had greater excess demand. In round 2, overall demand is the same as in round 1 at 3900 lots. No bidder has reduced demands. However, bidders did switch some quantity from one product to others. As a result, at the end of round 2 there is excess demand for all products, and so all products have higher prices in round 3. By round 9, supply and demand balance for four of the five products; only the 3-year product has excess demand. Thus, in round 10, only the 3-year product has a higher price. All products clear, and the auction ends, when the 3-year price reaches $7.85 and there is a reduction of 50, causing demand to match supply.

[18] Mandatory participation would still permit producers to withold supply from the auction in order to maintain higher prices, or to declare most, or even all, of their available production capacity as "interruptible" under current regulations, and hence avoid the requirement to participate in the auction. In addition, a cap on producers' reserve prices will likely need to be set by the regulatory authorities. Otherwise, producers would be able to withold supply simply by specifying very high reserve prices.

Figure 6. An auction with many products

2009 auction for delivery at Cusiana
all contracts start in 2010; lot = 100 MBTU/d; price = US$/MBTU

Excess demand
No excess demand

Round	Supply	1-year	2-year	3-year	4-year	5-year	Total
	Supply	600	500	400	400	400	2300
1	Price	$5.00	$5.00	$5.00	$5.00	$5.00	
	Demand	1200	800	300	700	900	3900
2	Price	$5.50	$5.40	$5.00	$5.40	$5.60	
	Demand	1000	900	600	600	800	3900
3	Price	$5.90	$5.90	$5.50	$5.80	$6.00	
	Demand	900	900	600	550	750	3700
...							
9	Price	$7.60	$7.80	$7.70	$7.90	$7.90	
	Demand	600	500	450	400	400	2350
10	Price	$7.60	$7.80	$7.85	$7.90	$7.90	
	Demand	600	500	400	400	400	2300

Cramton proposed that at the end of every round the auctioneer would report: 1) the excess demand for each product; and 2) the prices for the next round. He also considered a number of variations on the ascending clock auction, including: (i) rules for when a seller is also a buyer; (ii) allowing for priority for internal Colombian demand; and (iii) simultaneous auctions for transport capacity and gas. The latter option is discussed in Section 5 below.

4.2 Simultaneous Sealed-Bid Auctions

Until recently, there has been no practical sealed-bid auction design which allows bidders to express preferences for multiple, substitute goods. Therefore the simultaneous ascending clock auction has been the standard design for selling differentiated but substitutable products. In the past few years, however, both Milgrom (2009) and Klemperer (2010) have proposed sealed-bid auctions specifically designed for the sale of substitute goods: the "assignment auction" and the "product-mix auction" respectively. The product-mix auction proposed by Klemperer (2010) has already been adopted by the Bank of England for its regular auctions to supply liquidity to banks, against different types of collateral, and other central banks are also considering using this design. Since both these designs have a number of common characteristics, we will refer to them generically as "simultaneous sealed-bid auctions".

The basic idea of the simultaneous sealed-bid auction is to allow participants to bid on multiple substitute products simultaneously, expressing their preferences for the different goods at different prices in a sealed-bid format. The auctioneer then looks at all the bids submitted and chooses one (uniform) market-clearing price for each of the products on sale. Each bidder is awarded the product or products that give it the highest profit, given its bids and the final auction prices.

When bidders bid truthfully, a simultaneous sealed-bid auction finds the exact market-clearing price for each good, and each bidder is awarded its preferred quantities, given the actual prices of all goods. In other words, every bidder obtains the allocation that it would have chosen if it had known the auction prices in advance. So bidders should bid honestly — i.e. they should express their actual preferences — unless they expect to be able to affect the market clearing prices, which is unlikely if the number of bidders is large enough.

To determine the quantities on sale in the sealed-bid auction, each supplier may announce the quantity of each good that it is willing to produce and the minimum price it is willing to accept before the auction; or suppliers can make bids "to sell" in the auction specifying the different quantities of the various goods that they are willing to sell at different prices. This second procedure allows, for example, suppliers to produce and sell a relatively larger quantity of the goods that are more highly demanded in the actual auction.

Although they have many common characteristics, Milgrom's assignment auction and Klemperer's product-mix auction differ in a number of respects. The geometric representation that Klemperer uses seems easier to understand, and to implement, when only two different goods are being sold and when the seller wants to express complex preferences between these goods.[19] Milgrom's assignment auction has been especially designed to be easily adapted to a number of different contexts and allows bidders to express a wider range of preferences. For example, it allows bidders to specify budget limits for their bids, and various other constraints, under some conditions. Milgrom has developed software to implement both the assignment auction and the product-mix auction, with an arbitrary number of products, using a linear programming procedure to obtain integer allocations when bidders' demands and constraints are linear.

The main practical difference between the two simultaneous sealed-bid designs is the way in which bidders express their preferences (e.g., their demand curves for the products on sale). In each auction format, each bidder submits one or more sets of bids. In the product-mix auction, bids belonging to the same set are all for the same quantity and each refers to only one of the goods on sale. A bid in a set specifies the maximum price that the bidder is willing to pay for one of the goods; and the bids in a set are mutually exclusive, except for "marginal bidders" who are indifferent between the two goods at the market clearing prices and may be rationed. Basically, by submitting one set of bids a bidder declares the total quantity of the goods that it is willing to acquire with those bids, and by submitting different price-bids for the different goods the bidder expresses his relative valuation for the goods (i.e., at what price it is willing to substitute one good for another). Bids in a set represent alternatives for the auctioneer. After

[19] See Annex B for a geometric representation relating to the sale of firm and interruptible contracts.

observing all the bids, the auctioneer determines the auction prices for each good that clear the market and, from each set of bids offered by each bidder, the auctioneer accepts the one that gives the bidder the greatest surplus evaluated at the auction prices, or no bid if all bids yield negative surplus (i.e., if they are lower than the auction prices).

Similarly, in the assignment auction a set of bids by one bidder contains one bid for each of the goods on sale but, in contrast to the product-mix auction, each bid specifies both the maximum quantity of the good that the bidder is willing to acquire and the maximum price that he is willing to pay for the good. Moreover, bidders can choose a total quantity constraint for all bids in a set, thus specifying the maximum quantity of all goods that it is willing to acquire with that set of bids. As in the product-mix auction, this allows bidders to express their relative valuation for the goods on sale.

To compare the bidding procedures in the two auction designs, consider a bidder in an auction with 2 goods on sale, firm and interruptible gas supply contracts, who demands 10 units of contracts in total and is willing to pay up to v_F for firm contracts and up to v_I for interruptible contracts, where $v_F > v_I$. The sets of bids in the two auctions designs that allow the bidder to express its preferences are the followings:

Product-Mix Auction

Bid	**Product**	**Price**
1	Firm	v_F
2	Interruptible	v_I

Total Quantity: 10

Assignment Auction

Bid	**Product**	**Price**	**Quantity**
1	Firm	v_F	10
2	Interruptible	v_I	10

Limit on Total Quantity: 10

With both types of bids, the bidder is declaring that it is willing to acquire at most 10 units of either firm or interruptible contracts, but prefers firm contracts if they are more expensive than interruptible contracts by no more than $v_F - v_I$, while it prefers interruptible contracts if they are less expensive than firm contracts by at least $v_F - v_I$. Notice that, by submitting such a bid, the bidder is able to express the exact relative auction prices at which it is willing to substitute firm with interruptible contracts — i.e., when the price difference between the two contracts is $v_F - v_I$. In the following section, we describe the bidding procedures of the simultaneous sealed-bid auction in more details, by discussing various additional examples.

In an application like the sale of gas supply contracts, the difference in the procedures for submitting bids in the product-mix auction and in the assignment auction is not crucial, since both auction designs allow bidders to express preferences for substitute goods, and it should be relatively easy for bidders to familiarize themselves with any of them. So the two simultaneous sealed-bid designs are essentially equivalent.

4.2.1 Expressing Bidders' Preferences in the Simultaneous Sealed-Bid Auction

In order to describe in more detail how bidders can express their preferences in a simultaneous sealed-bid auction, we consider an example of a product-mix auction for the sale of gas supply contracts. We assume that there are only two types of contracts on sale, firm and interruptible, and consider three potential bidders.

Bidder 1 is a distribution company that: (a) needs 20 units of gas in firm contracts to serve the regulated market, regardless of the price; and (b) wants to buy up to another 10 units to serve unregulated customers in either firm or interruptible contracts, depending upon both the absolute and the relative prices of the two contracts. Specifically, for the additional 10 units bidder 1 is willing to pay up to v_F^1 for each firm contract and up to $v_I^1 < v_F^1$ for each interruptible contract. Therefore, bidder 1 prefers to acquire 10 additional units of firm over interruptible gas contracts so long as the price difference between the two types of contracts is less than $v_F^1 - v_I^1$.

To express these preferences, Bidder 1 can submit the following two sets of bids:

1. A bid for 20 units of firm contracts at an arbitrarily high price.
2. A set of 2 bids with:

Bid	**Product**	**Price**
1	Firm	v_F^1
2	Interruptible	v_I^1
	Total Quantity:	10

The first bid ensures that Bidder 1 always obtains exactly 20 units of firm gas contracts. The second set of bids ensures that Bidder 1 obtains the additional 10 units of gas contracts if and only if the price is lower than its willingness to pay. Which type of contracts it will actually obtain depends on the auction prices: Bidder 1 is awarded interruptible contracts if and only if the auction price of interruptible contracts is lower than the auction price of firm contracts by at least $v_F^1 - v_I^1$; otherwise it is awarded firm contracts, exactly as the bidder would have chosen had it known the relative prices in advance. This is because the auctioneer awards Bidder 1 the contract whose price is further away from its bid, i.e. the contract that provides the largest "margin" or "profit" to Bidder 1. So the auctioneer awards Bidder 1 firm contracts if and only if the difference between the prices of firm and interruptible contracts is lower than the difference between the bidder's bids for those contracts.

Bidder 2 is a thermal power station which needs 15 units of gas in firm contracts to serve the firm energy market, and is willing to pay at most 100 for each contract (since it can switch to normally more expensive fuel oil when gas prices are too high). Moreover, if the price of a firm contract is lower than 75, bidder 2 is willing to acquire 10 additional units of firm gas contracts. In order to express its preferences (i.e., its demand function for firm contracts), bidder 2 can simply submit 2 bids:

1. A bid for 15 units of firm contracts at price 100.
2. A bid for 10 units of firm contracts at price 75.

These bids ensure that Bidder 2 obtains 25 units of firm gas contracts if and only if the auction price of firm gas contracts is lower than 75, and that Bidder 2 obtains 15 units of firm gas contracts if and only if the auction price of firm gas contracts is higher than 75 and lower than 100. Bidder 2 acquires no contract if the auction price for firm contracts exceeds 100.

Bidder 3 is a trader that is willing to buy 30 units of either firm or interruptible contracts, depending upon both the absolute and the relative prices of the two contracts. Specifically, Bidder 3 is willing to pay up to v_F^3 for each firm contract and up to v_I^3 for each interruptible contract, but prefers to acquire interruptible contracts if they are cheaper than firm contracts by at least $v_F^3 - v_I^3$. To express its preferences, Bidder 3 can submit the following set of bids:

Bid	**Product**	**Price**
1	Firm	v_F^3
2	Interruptible	v_I^3
	Total Quantity:	30

This set of bids ensures that:

- Bidder 3 acquires 30 units of firm contracts if and only if the price of firm contracts is: (a) lower than v_F^3 and (b) lower than the price of interruptible contracts plus $v_F^3 - v_I^3$ (so that it obtains higher profit by acquiring firm rather than interruptible contracts);
- Bidder 3 acquires 30 units of interruptible contracts if and only if the price of interruptible contracts is: (a) lower than v_I^3 and (b) lower than the price of firm contracts minus $v_F^3 - v_I^3$ (so that it obtains higher profit by acquiring interruptible rather than firm contracts);
- Bidder 3 is not awarded any contract if and only if the prices of both firm and interruptible contracts are higher than the bidder's bids (so that it cannot obtain positive profit by acquiring any of the contracts).

This is exactly what the bidder would have chosen according to its preferences, given the auction prices.

This procedure for expressing bidders' preferences can be easily extended to more complex gas contract auctions. Suppose, for example, that gas contracts can be acquired from two alternative locations: G (Guajira) and C (Cusiana). In this case there are four different substitute contracts on sale in the auction: (1) firm gas contracts from location G; (2) firm gas contracts from location C; (3) interruptible gas contracts from location G; and (4) interruptible gas contracts from location C. Let p_i^j be the auction price of a contract of type i from location j, $j = G, C$ and $i = F, I$ (where F refers to firm gas contracts and I refers to interruptible gas contracts).

Consider again Bidder 3 and assume, for simplicity, that its valuations for firm and interruptible gas contracts are $v_F^3 = 80$ and $v_I^3 = 50$. Moreover, suppose that the transport costs that Bidder 3 has to pay for gas acquired from locations G and C are equal to c_G and c_C, respectively, with $c_G < c_C$. Therefore, Bidder 3 prefers to acquire a particular type of gas contract from location G rather than C if and only if the price difference between contracts from these two locations is lower than $c_C - c_G$.

In order to express its preferences for the four different types of contracts at different prices, Bidder 3 can now submit the following set of 4 bids:

Bid	**Product**	**Price**
1	Firm from G	$80 - c_G$
2	Interruptible from G	$50 - c_G$
3	Firm from C	$80 - c_C$
4	Interruptible from C	$50 - c_C$
	Total Quantity:	30

This set of bids ensures that Bidder 3 obtains exactly the type of contracts that it would have chosen given the auction prices. In other words, Bidder 3 obtains the type of contracts for which the difference between its price bid and the auction price is highest. For example, Bidder 3 is awarded 30 units of firm contracts from location G if and only if the price of this type of contracts — i.e., p_F^G — is: (a) lower than $80 - c_G$ (so that Bidder 3 obtains positive profit by acquiring that type of contracts); (b) lower than $p_I^G + (80 - 50)$ (so that Bidder 3 obtains higher profit by acquiring firm rather than interruptible contracts from location G); (c) lower than $p_F^C + (c_C - c_G)$ (so that Bidder 3 obtains higher profit by acquiring firm contracts from location G rather than location C); (d) lower than $p_I^C + (80 - 50) + (c_C - c_G)$ (so that Bidder 3 obtains higher profit by acquiring firm contracts from location G rather than interruptible contracts from location C).[20]

[20] In other words, letting b_i^j be the price bid by bidder 3 for a contract of type i from location j ($j = G, C$ and

It is straightforward to extend this procedure for expressing bidders' preferences to an auction with more than four types of contracts on sale — e.g. when contracts have different durations and/or refer to more than two alternative locations.

4.2.2 Choosing Prices in the Simultaneous Sealed-Bid Auction

After observing all bidders' bids, the auctioneer chooses auction prices equal to the market-clearing prices for all contracts on sale. That is, the auctioneer chooses prices such that, for each contract on sale, the total demand by bidders is equal to the total supply. Typically, if there are multiple market-clearing prices, the auctioneer chooses the lowest ones, but other criteria can also be used.[21]

The auctioneer awards to each bidder the type of contracts that provides it the highest margin, given its bids — i.e. it awards a bidder the type of contracts whose auction price is further below the bidders' price bid for that type of contract. The auctioneer awards no contract to a bidder if the auction price of each type of contract is higher than its price bid for that type of contract.

4.3 Comparison between Simultaneous Ascending and Sealed-bid Auctions

Both the simultaneous ascending auction (when the bid increments chosen by the auctioneer tends to zero) and the simultaneous sealed-bid auction find market-clearing prices and quantities for the goods on sale when bidders bid honestly in the sealed-bid auction and straightforwardly in the ascending auction (Milgrom, 2000). So the two auction formats are equivalent in these cases. In fact, bids in a simultaneous sealed-bid auction can be interpreted as the instructions provided to an agent who bids on behalf of a bidder in a simultaneous ascending auction, with the advantage that the bidding agent in the simultaneous sealed-bid auction does not make mistakes. So the simultaneous sealed-bid auction can be interpreted as a "proxy" simultaneous ascending auction.[22]

Nonetheless, the dynamic and the sealed-bid characteristics of the two types of auctions generate some notable differences for practical applications. Specifically, a simultaneous ascending auction has the following advantages, compared to a simultaneous sealed-bid auction:

1. **Information revelation and price discovery.** The simultaneous ascending auction

$i = F, I$), bidder 3 is awarded a contract of type i from location j if and only if $b_i^j - p_i^j \geq b_{i'}^{j'} - p_{i'}^{j'}$, for every $(i', j') \neq (i.j)$.

[21] A detailed procedure for selecting the auction prices will be specified in the auction rules.

[22] Notice that, in order to guide its bidding in a simultaneous ascending auction, a bidder should devise tables like the ones in Section 4.2 that express bids in a simultaneous sealed-bid auction. Therefore, preparing such tables to bid in a sealed-bid auction imposes no extra burden on bidders. On the contrary, to implement even the simple strategy described by the tables in an ascending auction, bidders would need to determine switches and quantity reductions in real time during the auction as prices change.

allows bidders to revise their bidding strategy, and possibly their own estimation of the values of the goods on sale, as other bidders' intentions and information are revealed during the course of the auction. This facilitates "price discovery" by bidders, which is impossible in a sealed-bid auction. Price discovery, however, is only relevant if the objects on sale have common value elements for bidders (e.g. if all bidders have roughly the same valuation for an object) and if bidders have private information about the objects' values.[23]

2. **Experience and applications.** The simultaneous ascending design has been tested in many practical applications over the last 10 years, and is familiar in Colombia. By contrast, the simultaneous sealed-bid auction is a novel design which is much less tested.

3. **Revelation of preferences**. The simultaneous ascending auction does not require winners to reveal their valuations for the goods on sale (since the auction terminates when the prices reach the valuations of losing bidders), while the sealed-bid auction does. There are situations in which high-value bidders may be unwilling to reveal their valuations truthfully in a sealed-bid auction.

By contrast, the simultaneous sealed-bid auction has the following advantages, compared to a simultaneous ascending auction:

1. **Time and speed**. The simultaneous ascending auction requires several rounds of bidding and possibly a considerable amount of time to reach a conclusion. This process is time-consuming and costly, and requires bidders' attention during the whole course of the auction. The time required and the costs for bidders may reduce participation in the auction. By contrast, the simultaneous sealed-bid auction is instantaneous, thus saving time and transaction costs for all parties involved.

2. **Accuracy for the auctioneer**. The simultaneous ascending auction requires the auctioneer to make guesses when setting the prices of different goods between rounds while, at the same time, it allows bidders to switch demand between products. But an auctioneer cannot always determine the correct price increments that would keep the quantity of the various goods demanded by bidders properly balanced. So the ascending auction cannot guarantee finding the exact market clearing prices in a practical application. (See the examples in Annex A). A partial solution to these potential problems is to introduce fine-tuned activity and switching rules, and intraround bidding. By contrast, the simultaneous

[23] Notice, however, that it is possible to allow price discovery in a simultaneous sealed-bid auction, by using a 2-stage design. Such a design would reduce the differences between the two auction format, since it would introduce a dynamic element in the simultaneous sealed-bid auction.

sealed-bid auction is a straightforward and ready-to-use mechanism that always finds the exact market-clearing prices.

3. **Accuracy for bidders**. The simultaneous sealed-bid auction eliminates the need for bidders to make real-time demand calculations, bid decisions and bid entry, that may generate mistakes. Moreover, it allows bidders to express their precise preferences within a relatively simple procedure. By contrast, due to the discreteness of the price increments, the simultaneous ascending auction introduces more restrictions on the preferences that bidders can express.

4. **Collusion**. Collusion (whether implicit or explicit) among bidders is typically easier in an ascending auction than in a sealed-bid auction, because bidders can observe and respond to competitors' behavior in an ascending auction. Hence it is easier for them to coordinate their action and manipulate prices.[24] For example, coordinated demand reduction and predatory strategies aimed at reducing the auction prices are much easier in ascending auctions. The reason is that, in an ascending auction, bidders may observe a cooperative reduction in demand by their competitors and respond by reducing their own demand, and they may punish a bidder that fails to cooperate (for example by bidding aggressively on a good that the bidder is especially interested in acquiring).[25] Of course, the information revealed during an ascending auction may be reduced to make collusion harder. But reducing the information revealed also reduces the main advantage of an ascending auction: the ability to facilitate price discovery. Basically, providing bidders with more information during the auction creates a trade-off: more information may be useful for bidders both to refine their valuations and to (implicitly or explicitly) collude.

Summing up, the simultaneous sealed-bid auction is more robust to collusion and is a much faster procedure to sell substitute products than the simultaneous ascending auction. However, it does not allow bidder to revise their valuation after observing the behavior of their competitors during the auction. Hence, the simultaneous sealed-bid auction is well suited for situations in which bidders have a clear understanding of the quantity of the different goods that they are willing to acquire at different prices, before participating in the auction.

A final consideration is the following. By providing information on the likely outcome of the auction during the bidding, the simultaneous ascending auction allows bidders to concentrate their efforts in discovering (or obtaining a more precise estimate of) their valuations only for

[24] In the Colombian gas market, collusion does not appear to be a major concern from the demand-side of the market. But since gas production is extremely concentrated in Colombia, collusion could be a major concern if producers are allowed to submit "sell bids" in the auction.

[25] More generally, bidders can make bids that signal threats and offers to other bidders.

the goods that they are likely to win. This characteristic of the simultaneous ascending auction could save time and resources for bidders and is especially valuable when a large number of goods are on sale, since in this case it could be very costly for bidders to determine their relative valuations for all goods. The drawback of this is that bidders may choose to wait until after the start of the auction to produce effort to determine their relative valuations for some of the good on sale. For example, a bidder may decide that it is particularly interested in acquiring one of the goods, and refrain from determining before the auction at what relative prices it should switch its demand to substitute goods (ignoring the fact that it may not be able to acquire its most preferred good).

But estimation made during the course of the auction may turn out to be imprecise due to time constraints, potentially leading to inefficiency. By contrast, a sealed bid auction forces bidders to determine their relative valuations for all the goods that they are possibly interested in acquiring, before bidding in the auction, when they have all the time necessary to produce precise estimations. Bidding in a sealed-bid auction may be a discipline device for bidders, which is especially valuable when there are relatively few goods on sale, so that determining the precise relative valuations for all of them is not too costly.

5 Further Issues in Auction Design

This section considers a number of other issues in the auction design. These are:

- should the auctions set a Colombia-wide price for gas, or should there be separate (but simultaneous) auctions for different fields?
- should purchase of gas supplies and transport contracts be coordinated?
- should the sale of gas in conditional firm or interruptible contracts be included in the primary auctions?
- should the auctions be simultaneous or sequential for different types of contract?

5.1 Colombia-wide or Location-specific Auctions

A key issue which needs to be addressed is whether contracts in the primary auctions should distinguish supplies by location, i.e. specify the field from which the gas is to be delivered. This would mean that there would effectively be a separate but simultaneous auction for each participating field, such as Guajira, Cusiana and La Creciente, and buyers would bid for each separately, likely resulting in different prices for gas supplied from different fields.

An alternative would be to treat all gas supplies offered in the auctions *as if* they originated in the same location, and allow the auction to establish a single, Colombia-wide price for all gas contracts of a given type (e.g. firm or interruptible) and duration (e.g. one-year or three-year contracts).[26]

In the former case, gas contracts in the simultaneous, field-specific auctions would specify the location or field, but not the company, so a buyer purchasing gas from the Guajira fields would be awarded contracts from both Chevron and Ecopetrol in proportion to amounts offered by each company.

In the latter case, the contracts in the auction would specify neither the company nor location, so a successful bidder would be allocated gas *ex post* from different fields as well as from different companies. For example, a successful bidder in Cartagena could be awarded gas from both the Guajira and La Creciente fields, while a bidder in Bogota would be awarded contracts from producers in both the Guajira and Cusiana fields. An example of such an allocation scheme is provided in Table 5.1 below.

Table 5.1 Post Auction Allocation Example

				Demand/Allocation	
Field	**Company**	**GBTUD**	**Share**	**Interior**	**North Coast**
				600	*300*
La Guajira	Ecopetrol	400	66.67%	233.33	166.67
	Chevron	200	33.33%	116.67	83.33
Total		**600**	**100%**	**350**	**250**
Cusiana	Equion	200	80%	200	
	Total/Tepma	50	20%	50	
Total		**250**	**100%**	**250**	**0**
La Creciente	Pacific Rubiales	**50**	**100%**	**0**	**50**
TOTAL		**900**		**600**	**300**

In the example, Interior demand at the auction closing (i.e. market-clearing) price is 600 GBTUDs and North Coast demand is 300 GBTUDs. All of the supply from La Creciente (50 GBTUDs) must be allocated to North Coast demand, and all of the Cusiana supply (250 GBTUDs) must be allocated to Interior demand. The remaining North Coast demand must be allocated contracts from Guajira (250 GBTUDs in total) and from each company according to their market share. The remaining Interior demand is allocated contracts from the Guajira producers in proportion to their market shares.

Since by definition total supply equals total demand at the auction closing price, such an allocation is always possible so long as no transmission constraints are violated. The most

[26] This discussion assumes a deregulated Guajira price.

relevant such constraints are: (i) that North Coast demand at the clearing price not exceed the total supply of gas available from La Creciente and Guajira;[27] and (ii) that the allocation of Guajira gas to Interior demand not exceed the capacity of the Ballena-Barrancabermeja-Bogota pipeline. If either of these constraints is likely to bind, an auction which ignores location would be complex to implement and probably not practical.

Transportation costs Assuming that a Colombia-wide auction is feasible, since the contracts would not specify location in advance, bidders would face some uncertainty concerning the transportation costs they will incur from any winning bid. For a North Coast bidder the proportion of gas they will receive from La Creciente and Guajira will not be known with certainty, and an Interior bidder will face similar uncertainty concerning the amount of Guajira versus Cusiana gas they will receive. If the total North Coast demand can be predicted with reasonable accuracy, however, then the uncertainty will be small. If North Coast demand were known in advance with certainty, for example, then once the total supply from each field was known, each bidder could predict with certainty exactly the proportions of gas from each field they would receive from any winning bid. Typically however, bidders will not know this and the resulting uncertainty over transport costs will potentially lead to more cautious bidding behavior in the auctions.

The importance of this depends entirely on the materiality of the differences in transportation costs in shipping gas from one field or another to the place of final consumption. The Table 5.2 below contains the necessary information.

Table 5.2 assumes a uniform auction price for gas of \$5.5 (US) per MBTUD. For demand centres Barrancabermeja, Bogota, Medellin and Cali the table shows differences and percentage differences in total cost per MBTUD from being allocated gas contracts from La Guajira versus Cusiana. For demand centres Barranquilla, Cartagena and Cerromatoso the differences are between being allocated gas contracts from La Guajira versus La Creciente.

[27] While unlikely in the immediate future, this may become more of an issue as the production of the Guajira fields declines over time. Current demand on the North Coast is less than 400 GBTUDs, capacity on the Promigas system is 540 GBTUDs, and current production from the two fields exceeds 750 GBTUDs.

Table 5.2 Gas and Transport Costs Differences per MBTUD (US)

Production Field	Demand	Gas cost	Transport cost	Total	Cost diff.	Percentage diff.
From La Guajira						
to:	Barrancabermeja	5.5	1.32	6.82		
	Bogota	5.5	2.25	7.75		
	Medellin	5.5	2.46	7.96		
	Cali	5.5	3.24	8.74		
	Barranquilla	5.5	0.53	6.03		
	Cartagena	5.5	0.65	6.15		
	Cerromatoso	5.5	1.12	6.62		
From Cusiana						
to:	Barrancabermeja	5.5	1.4	6.9	−0.08	−1.19%
	Bogota	5.5	1.67	7.17	0.58	7.48%
	Medellin	5.5	2.39	7.89	0.07	0.88%
	Cali	5.5	2.81	8.31	0.43	4.87%
From La Creciente						
to:	Baranquilla	5.5	0.74	6.24	−0.21	−3.52%
	Cartagena	5.5	0.61	6.11	0.04	0.62%
	Cerromatoso	5.5	0.52	6.02	0.6	9.03%

While some of these transportation cost differences seem to be material, in our consultations with the industry there appeared to be general agreement that the auction products should be non-location specific, and that the auctions should establish Colombia-wide prices for gas contracts of different types.

A trading hub solution? Sections 3.4 and 3.5 of "Designing and Structuring the Secondary Market, Short-term Markets and their Management Mechanisms: Task 4 Report," describe possible ways in which either a physical hub or a "virtual trading point" could be established in Colombia. The latter is essentially equivalent to trading non-location specific contracts as described in this section. If any such scheme was adopted it could equally well be applied to the upstream auctions.

5.2 Coordination of Gas and Transport Capacity

Participants in the primary auctions require both gas and transport from the supply delivery point to the buyer's demand location. Assuming that the pipeline network is uncongested, the required capacity on the transport network will always be available. Indeed, it would be possible to simply allocate the necessary transport capacity to each winning purchaser in the auction at the regulated transport prices.

If pipeline capacity is scarce however, buyers will face a coordination problem, and have two options: first buying gas in the auction and then securing pipeline capacity, or buying transport capacity first and then participating in the auction for gas. Both of these options involve a "chicken-and-egg" problem for the buyer.

One possible solution would be to auction scarce transport capacity simultaneously with gas purchase contracts, albeit at the expense of complicating the primary gas auctions considerably. Transport capacity could be offered both by transport system operators (TSOs), and by shippers who hold long-term transport contracts.[28] This would have the merit of ameliorating the coordination problem for gas purchasers, although the degree of amelioration would depend upon whether or not "package bids" were allowed (i.e. allowing the purchase of a quantity of gas to be made contingent upon the simultaneous purchase of transport capacity). It would also allow market-based prices to be set for scarce transport capacity.[29]

From our discussions with the Colombian industry last year (see the discussion in Harbord 2010), however, gas buyers appeared to face a "one-way" coordination problem in the sense that they were always able to purchase firm transport capacity, but firm gas supply contracts were unavailable, or in short supply. Given this, it may be that the introduction of primary gas auctions will resolve this issue for purchasers without the need to introduce any specific market mechanism for coordinating gas and transport capacity purchases.

5.3 Sale of Conditional Firm Contracts in the Auctions

Gas-fired power plants in the interior of Colombia purchase large quantities of firm gas contracts in order to participate in the firm energy market and receive "reliability" payments. Their demand for firm gas supplies amounts to possibly 45% of the total available supply of gas in Colombia, and this gas must be resold to other consumers in conditional firm or interruptible contracts.[30] Hence the gas-fired power plants face a "coordination" problem in the sense that they would benefit from being able to make their purchases of firm gas contracts in the primary auctions contingent upon selling equal quantities of gas in conditional firm or interruptible contracts.

Given that a significant percentage of the firm gas offered in the primary auctions is going to

[28] For example, Isagen is currently holding daily auctions for 24 hour firm gas contracts and simultaneously offering to sell transport capacity to each winning bidder.

[29] Under the current regulatory framework for pricing transport capacity, the more congested a pipeline becomes, the lower are the regulated average cost based charges (see Harbord and von der Fehr 2010 for a discussion). Using auctions to allocate pipeline capacity would ensure that shippers faced the market-determined opportunity cost of their usage decisions and provide better information for location and investment decisions.

[30] Conditional firm contracts are gas supply contracts which are interruptible only when the spot market price of electricity exceeds the regulated "scarcity price", when gas-fired generators may be called upon to operate under their firm energy market obligations. See Milgrom *et al* (2011) for a discussion of these contracts, and Cramton and Stoft (2007) and Harbord and Pagnozzi (2008) for more detail on the Colombian firm energy market.

be purchased by gas-fired generators, and subsequently offered for resale in the secondary market, it might be valuable to allow for this resale to occur simultaneously with the primary gas supply contracts. Clearly the gas supply contracts offered by the power plants in the secondary market are close substitutes for the primary gas contracts to be sold in the auctions for many other gas purchasers. Economic theory suggests that it is better if such close substitutes are offered in the same auction. There are a number of possible ways of doing so.

In the open ascending (clock) auction and in the simultaneous sealed-bid auction, gas-fired power plants could simply participate as both buyers and sellers, offering to purchase firm gas contracts from producers whilst simultaneously offering to sell conditional firm contracts.

In a simultaneous sealed-bid auction, the power plants could use "swap" bids, buying firm gas and selling an equal quantity of conditional firm. Alternatively, power plants could commit to buy firm gas at whatever price clears the firm market and offer conditional firm contracts in the same auction in known quantities and at some pre-specified reserve price, as substitutes for the firm contract. Compared to swap bids, these do not let the reserve price of conditional firm contracts depend on the price of the firm contracts.

Another (complementary) possibility would be for the upstream gas producers to sell conditional firm contracts in the auctions.[31] To do so they would need to sell two additional contract types: (i) a conditional firm contract to distributors, industrial or other buyers who are willing to risk supply interruptions when the electricity spot price exceeds the regulated scarcity price; and (ii) an "option" contract to gas-fired power plants giving them the right to purchase the gas when called upon to operate under their firm energy market obligations.

Either of these alternative ways of selling conditional firm contracts in the upstream auctions create issues which will need to be resolved, in particular the extent to which either power plants or producers would be able to link or "package" the purchase or sale of one type of contract with another.

5.4 Simultaneous versus Sequential Auctions

A final issue is whether different types of gas contracts, such as firm and interruptible contracts and contracts of different durations, should be sold in separate, sequential auctions, or whether they should be sold together in a combined, simultaneous auction. Separating the sale of different contracts has the advantage of simplifying each auction; participants can concentrate their attention on the particular contract for sale and bid according to their willingness to pay for this particular contract. When there is no direct relation between different types of contracts - in particular, bidders' willingness to pay for a particular contract does not depend directly on their holding of other contracts - this would seem the natural choice.

[31] Some gas producers already sell limited quantities of these contracts in the primary market.

However, when bidders' valuations of particular contracts depend on their overall contract portfolio, they will benefit from being able to optimize their holdings of different contracts simultaneously. For example, a buyer of gas may want to optimize the shares of firm and interruptible contracts in a contract portfolio covering a given volume of gas, and in order to do so effectively, the buyer would want to adjust contract positions of the two types of contracts in parallel according to relative prices. This can only be done if the different types of contracts are sold in a simultaneous auction.

Gas contracts may be related in different ways. Two gas contracts are said to be substitutes if the willingness to pay for one of the contracts is less when the bidder already holds the other contract than when he does not; conversely, two gas contracts are said to be complements if the willingness to pay for one of the contract is greater when the bidder already holds the other contract than when he does not. Contracts with different durations with the same start date are likely to be substitutes, as noted in Section 3 above, while contracts with different start dates are likely to be complements.

The choice between sequential and simultaneous auctions is a trade off between simplifying individual auctions and allowing bidders flexibility to optimize their contract portfolios.

6 Conclusions and Recommendations

In this first report it is not our intention to reach firm recommendations or conclusions on any of the issues considered. Rather, our purpose is to delineate and describe the various options available and to come to conclusions only after further detailed consultations with the industry and the CREG. We have reached some preliminary conclusions on the auction products and frequency, however.

Auction Products and Frequency

- **lot size:** 10 - 20 MBTUDs
- **contract durations:** one-year and five-year contracts
- **contract start dates:** six months or one year from the date of the each auction
- **contract indexation:** the Colombian Producer Price Index (IPP) or other suitable index for three-year contracts
- **auction frequency:** annual

These recommendations may be revised following further consultations with the CREG and the Colombian industry.

Auction Design Both the simultaneous ascending clock auction and the simultaneous sealed-bid auction are viable alternatives for the auction design. We have discussed the pros and cons associated with each. A recommendation will only be made after the May workshops in Bogota and further consultations.

Other Issues The auction design issues discussed in Section 5 require further consideration, and recommendations will only be made after the May workshops and consultations.

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A Potential Problems with the Simultaneous Ascending Clock Auction

A simultaneous ascending-good clock auction can terminate with excess supply on a good, when a bidder switches demand and causes total demand on a good to be lower than supply. Examples 1 and 2 highlight the potential problems caused by this issue. Intraround bidding in ascending auctions reduces the problem created by discrete price increments, but still does not allow a bidder to report the precise relative prices at which he wants to reduce demand on a good, or switch demand between goods. Example 3 discusses this issue. Example 4 shows why preventing a bidder from switching when this produces excess supply may result in an inefficient allocation.

Suppose there are 2 substitute goods on sale: A and B. The supply of each of the goods is equal to 100. Between rounds, the price of a good is increased by 1 if and only if there is excess demand for that good. Let p_i be the price of good i. The activity rule allows bidders to switch demand between goods (if prices change), while bidders cannot increase their total demand for the two goods. We assume that bidders bid straightforwardly (i.e. truthfully, based on their preferences). We restrict prices to be integers, for simplicity, apart from Example 3 where we also consider intraround bidding .

A.1 Example 1

Consider 3 bidders with the following preferences:

- Bidder 1 is willing to acquire 10 units of either good A or good B, and has a per-unit value of 24.5 for good A and 20 for good B.[32] Hence, bidder 1 is indifferent between A and B when $p_A - p_B = 4.5$, but prefers to win A if $p_A - p_B < 4.5$ (and prefers to win B if $p_A - p_B > 4.5$).
- Bidder 2 demands 10 units of B and has a per-unit value of 5.5.
- Bidder 3 demands 10 units of B and has a per-unit value of 8.5.

We also assume there are a number of other bidders with a total demand for A of 95 and a total demand for B of 85 which do not vary with prices. Suppose that, in round $t-1$, prices are $p_A = 9$ and $p_B = 5$ and the total demand (including all active bidders) for both A and B is 105. Because there is excess demand on both goods, in round t the price of A increase to 10 and the price of B increase to 6, so that bidder 1 is bidding on A and bidder 2 reduces his demand of 10 units on B. Hence, the total demand for A is 105 and the total demand for B is 95. Since there is excess demand on good A, but not on good B, in round $t+1$ the price of A goes up to 11, while the price of B stays at 6. This induces bidder 1 to switch demand to B (since now the price difference is equal to 5). Hence, the total demand for A is 95 and the total demand for B is 105.

Since there is now excess demand on B, but not on A, in round $t+2$ the price of B goes up to 7, while the price of A stays at 11. This induces bidder 1 to switch demand to A (since now the price difference is equal to 4). Hence, the total demand for A is 105 and the total demand for B is 95. But now there is again excess demand on A, but not on B (as in round t). So in round $t+3$ the price of A goes up to 12, while the price of B stays at 7. This induces bidder 1 to switch demand to B. Hence, the total demand for A is 95 and the total demand for B is 105.

And so on ... Bidder 1 continues to switch demand between goods, as the price difference changes, creating excess demand on only one good at a time and inducing the price of that good to increase. In round $t+6$, the price of B reaches 9 and at that point bidder 3 reduces his

[32] The per-unit value of a good represents the maximum willingness to pay of the bidder for each unit of the good.

demand of 10 units on B (while at the same time bidder 1 switches demand to A). Hence, the total demand for A is 105 and the total demand for B is 85.

Finally, since there is excess demand on good A, but not on good B, in round $t + 7$ the price of A goes up to 14, while the price of B stays at 9. This induces bidder 1 to switch demand to B again. Hence, the total demand for A is 95 and the total demand for B is 95. There is no more excess demand on any good, and the ascending auction terminates with excess supply on both goods.

Round	p_A	p_B	Demand for A	Demand for B	Comment
...	...	...	...	...	
$t-1$	9	5	105	105	
t	10	6	105	95	Bidder 2 reduces demand on B by 10
$t+1$	11	6	95	105	Bidder 1 switches 10 units to B
$t+2$	11	7	105	95	Bidder 1 switches 10 units to A
$t+3$	12	7	95	105	Bidder 1 switches 10 units to B
$t+4$	12	8	105	95	Bidder 1 switches 10 units to A
$t+5$	13	8	95	105	Bidder 1 switches 10 units to B
$t+6$	13	9	105	85	Bidder 1 switches 10 units to A and bidder 3 reduces demand on B by 10
$t+7$	14	9	95	95	Bidder 1 switches 10 units to B

To eliminate excess supply, prices could be reduced and bidders could be rationed after the ascending auction terminates. But, in this example, to eliminate excess supply the prices of both goods should be substantially reduced, to 9 and 5 respectively. These are also the market-clearing prices.[33] By contrast, preventing bidders from switching demand away from a good when this would result in excess supply on that good may not be advisable, since it could induce an inefficient allocation of the goods on sale (see Example 4).

The potential problem described in this example is created by bidders' ability to switch demand among products in a clock auction. A more restrictive activity rule (e.g., requiring demand to weakly decrease on each good) would eliminate it, but it may lead to an inefficient allocation.

A.2 Example 2

Consider 4 bidders with the following preferences:

[33] Observe that if prices are not restricted to be integers (as in a simultaneous sealed-bid auction) and, when indifferent, a bidder can be rationed or his demand can be split between the two goods, then the market clearing prices are 10 and 5.5 respectively. Bidder 2 is not allocated any units and bidder 1 is allocated 5 units of A and 5 units of B.

- Bidder 1 demands 20 units of the two goods. He always prefers to win 20 units of A if $p_A < 11$, but he prefers to win 20 units of B if $p_A \geq 11$ and $p_B < 11$. Bidder 1 does not want to acquire any good if $p_A \geq 11$ and $p_B \geq 11$. (These preferences may arise if, for example, bidder 1 has a much higher valuation for A than for B, but faces a total budget constraint equal to $11 \times 20 = 220$. But notice that, in contrast to the preferences in example 1, these preferences are not linear.)

- Bidder 2 demands 20 units of B if and only if $p_B < 7$ (i.e., he has a per-unit value of $7 - \varepsilon$).

- Bidder 3 demands 20 units of B if and only if $p_B < 8$ (i.e., he has a per-unit value of $8 - \varepsilon$).

- Bidder 4 demands 10 units of B if and only if $p_B < 9$ (i.e., he has a per-unit value of $9 - \varepsilon$).

Suppose that, in round t, $p_A = 10$ and $p_B = 6$, so that bidder 1 is bidding on A. The total demand for A is 115 and the total demand for B is 130. Since there is excess demand on both goods, in round $t+1$ the price of A goes up to 11 and the price of B goes up to 7. This induces bidder 1 to switch his demand of 20 units to B and bidder 2 to reduce his demand of 20 units on B. Hence, in round $t+1$ the total demand for A is 95 and the total demand for B is 130.

Since now there is excess demand on good B, but not on good A, in round $t+2$ the price of B goes up to 8 and the price of A stays at 11. This induces bidder 3 to reduce his demand of 20 units on B (while bidder 1 still bids on B). Hence, in round $t+2$ the total demand for A is 95 and the total demand for B is 110.

Since there is still excess demand on good B, but not on good A, in round $t+3$ the price of B goes up to 9 and the price of A stays at 11. This induces bidder 4 to reduce his demand of 10 units on B (while bidder 1 still bids on B). Hence, in round $t+3$ the total demand for A is 95 and the total demand for B is 100. So the auction terminates with excess supply of A.

The following table reports the outcomes of the various rounds.

Round	p_A	p_B	Demand for A	Demand for B	Comment
...	...	...	...	...	
t	10	6	115	130	
$t+1$	11	7	95	130	Bidder 1 switches 20 units to B and bidder 2 reduces demand on B by 20
$t+2$	11	8	95	110	Bidder 3 reduces demand on B by 20
$t+3$	11	9	95	100	Bidder 4 reduces demand on B by 10

To eliminate excess supply, prices could be reduced and bidders could be rationed after the ascending auction terminates. Notice that, given the bidders' preferences, the "market-clearing prices" are $p_A = 10$ and $p_B = 7$. So the prices of A and B can be both reduced to 10 and 7, respectively. Then the total demand for A is 115 (since bidder 1 prefers to acquire A) and the total demand for B is 110. (Notice that reducing the prices to 10 and 8 would not eliminate excess supply on B.) To avoid complaints by other bidders, only bidder 1 should be rationed on good A; and only bidders 3 and 4 should be rationed on good B.

However, the problem is that in an ascending auction the auctioneer does not know bidders' demand when $p_A = 10$ and $p_B = 7$, because there was no round in the auction in which $p_A = 10$ and $p_B = 7$, and the auctioneer does not know bidders' preferences. To eliminate excess supply, the auctioneer can still reduce prices to the levels in round t — i.e., $p_A = 10$ and $p_B = 6$. But in this case the ascending auction does not find the exact market clearing prices.

A.3 Example 3

To reduce the problem created by the discreetness of price increments between rounds, intraround bidding can be introduces in simultaneous ascending auctions. However, intraround bids may still not allow bidders to express their preferences fully. In particular, intraround bids do not generally allow a bidder to indicate the precise relative price at which it is willing to switch demand between goods.

Consider, for example, a bidder who is indifferent between acquiring 10 units of good A or 10 units of good B when $p_A - p_B = 2.4$, but prefers to acquire 10 units of good A if and only if $p_A - p_B < 2.4$. Suppose that the current auction prices of A and B, at the beginning of a round, are equal to 9 and 7, respectively. The prices at the end of the round are 10 and 8, respectively. So the bidders prefer to bid for good A at the start-of-round prices (9 and 7), and at the end-of-round prices (10 and 8). But the bidder would prefer to bid on good B if, for example, the price of A is 9.8 and the price of B is 7.2. Even with intraround bidding, the bidder cannot express its preferences.

Moreover, suppose instead that in one round the price of good A increases from 9 to 11 while the price of good B increases from 7 to 8. (A different price increase for two goods may be due to a difference in the amount of excess demand on the two goods, that induces the auctioneer to try to balance excess demands among goods.) In this case, at the start-of-round prices (9 and 7) the bidder prefers to bid for good A, while at the end-of-round prices (11 and 8) it prefers to bid for good B. However, the bidder cannot indicate the precise point at which it is willing to switch demand between A and B, even with intraround bidding, since this depends on the prices of both goods.

The simultaneous sealed-bid auction solves this issue by allowing bidders to express their preferences for any price of all goods on sale.

A.4 Example 4

Suppose now there are 3 substitute goods on sale, A, B and C, and consider 4 bidders with the following preferences:

- Bidder 1 is willing to acquire 20 units of either A or B, and has a per-unit value of $v_A^1 = 18$ for A and $v_B^1 = 20$ for B. Hence, bidder 1 is indifferent between A and B when $p_B - p_A = 2$, but prefers to acquire B if $p_B - p_A < 2$ (and prefers to acquire A if $p_B - p_A > 2$).

- Bidder 2 is willing to acquire 20 units of either B or C, and has a per-unit value of $v_B^2 = 16.5$ for B and $v_C^2 = 20$ for C. Hence, bidder 2 is indifferent between B and C when $p_C - p_B = 3.5$, but prefers to acquire C if $p_C - p_B < 3.5$ (and prefers to acquire B if $p_C - p_B > 3.5$).

- Bidder 3 is willing to acquire 10 units of either A or C, and has a per-unit value of $v_A^3 = 11$ for A and $v_C^3 = 17.5$ for C. Hence, bidder 3 is indifferent between A and C when $p_C - p_A = 6.5$, but prefers to acquire C if $p_C - p_A < 6.5$ (and prefers to acquire A if $p_C - p_A > 6.5$).

- Bidder 4 is willing to acquire 10 units of C and has a per-unit value of $v_C^4 = 15.5$.

Suppose that, in round t, $p_A = 8$, $p_B = 10$ and $p_C = 14$, and that bidder 1 is bidding on A, bidder 2 is bidding on B and bidder 3 is bidding on C. The total demand for A is 110, the total demand for B is 100, and the total demand for C is 100. Since there is excess demand only on good A, in round $t+1$ the price of A goes up to 9. This induces bidder 1 to switch his demand of 20 units from A to B. Hence, in round $t+1$ the total demand for A is 90, the total demand for B is 120, and the total demand for C is 100.

Since there is excess demand only on good B, in round $t+2$ the price of B goes up to 11. This induces bidder 2 to switch his demand of 20 units from B to C. Hence, in round $t+1$ the total demand for A is 90, the total demand for B is 100, and the total demand for C is 120. Since there is now excess demand only on good C, in round $t+3$ the price of C goes up to 15 and in round $t+4$ the price of C goes up to 16. This induces bidder 3 to switch his demand of 10 units from C to A, and bidder 4 to reduce his demand of 10 units on C. Hence, the total demand for all goods is 100 and the auction terminates.

Round	p_A	Demand for A	p_B	Demand for B	p_C	Demand for C	Comment
t	8	110	10	100	14	100	
$t+1$	9	90	10	120	14	100	1 switches demand from A to B
$t+2$	9	90	11	100	14	120	2 switches demand from B to C
$t+3$	9	100	11	100	15	120	
$t+4$	9	100	11	100	16	100	3 switches 10 units to A and 4 reduces demand on C by 10

If bidder 1 is prevented from switching his demand in round t because this would generate excess supply of good A, the auction terminates in round t instead (with bidder 1 winning 10 units of A). This would reduce efficiency and the seller's revenue. The reason is that allowing a bidder to switch demand away from one good causes the prices of other goods to change, and this causes other bidders to reallocate their demand according to the new relative prices; preventing this may induce inefficiency. Indeed, in our example, the difference between the winning bidders' total valuations in round $t+3$ and in round t is equal to

$$\left[\left(10 \times v_A^3\right) + \left(20 \times v_B^1\right) + \left(20 \times v_C^2\right)\right] - \left[\left(10 \times v_A^1\right) + \left(20 \times v_B^2\right) + \left(10 \times v_C^3\right) + \left(10 \times v_C^4\right)\right] =$$

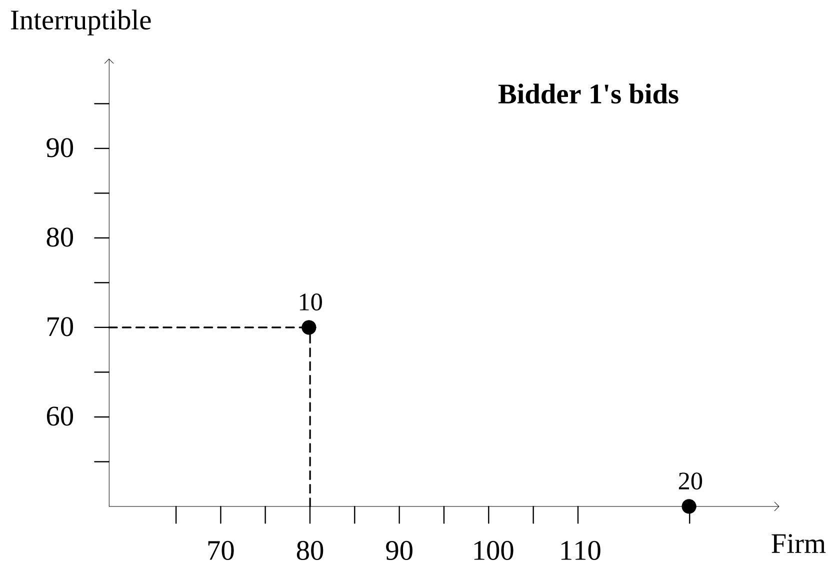
$$[(10 \times 11) + (20 \times 20) + (20 \times 20)] - [(10 \times 18) + (20 \times 16.5) + (10 \times 17.5) + (10 \times 15.5)] =$$

$$910 - 840 = 70.$$

B Geometric Representation of the Product-Mix Auction

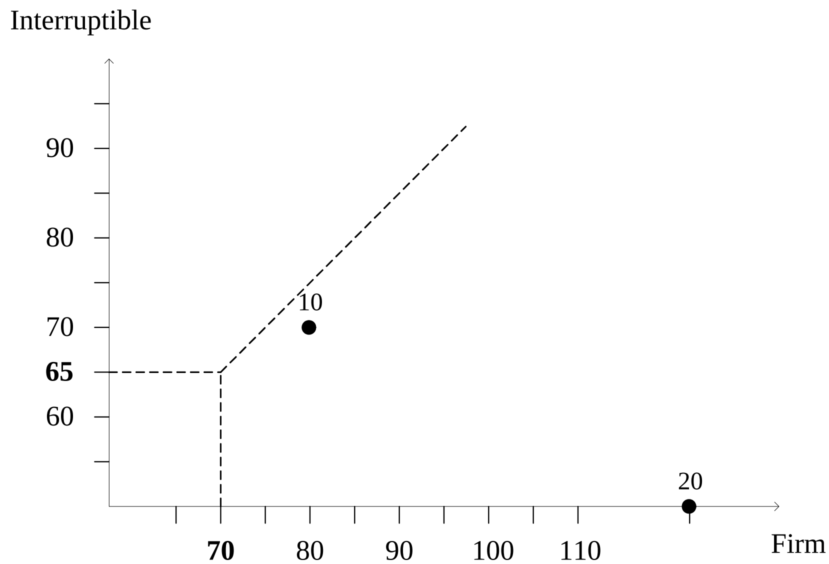
In this section, we provide a simple geometric representation of the product-mix auction. Consider an auction with only two products on sale: firm and interruptible gas contracts. We assume that the total supply of firm gas contracts is 90 units and the total supply of interruptible gas contracts is 60 units.

Consider bidder 1 described in Section 4.2.1 and assume that $v_F^1 = 80$ and $v_I^1 = 70$. The two sets of bids that express bidder 1's preferences can be represented in the following picture, where the price of firm gas in on the horizontal axis and the price of interruptible gas in on the vertical axis. Each dot represents one of the bids, and the number on each dot is the quantity associated to the bid.

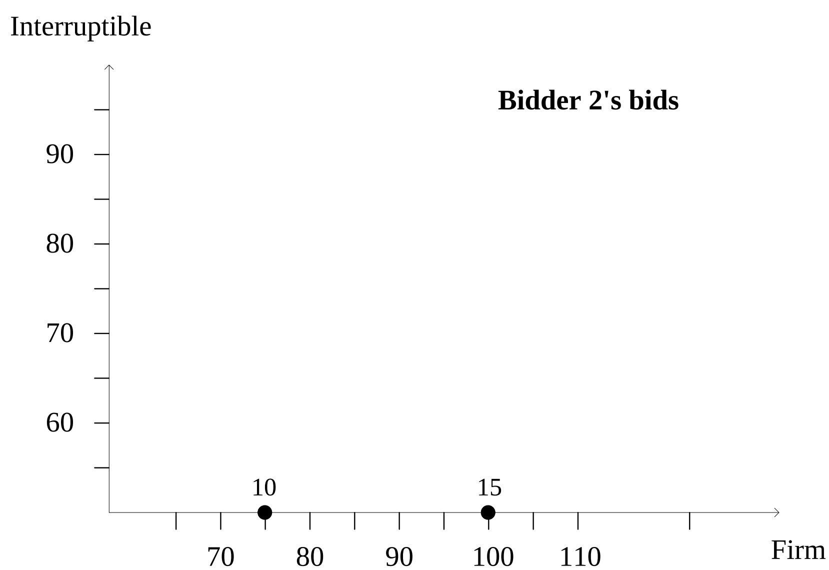


With the two bids described, bidder 1 is indicating that he is willing to acquire 20 units of firm contracts at any price and that, in addition, it is willing to acquire either 10 units of firm contracts at price 80, or 10 units of interruptible contracts at price 70.

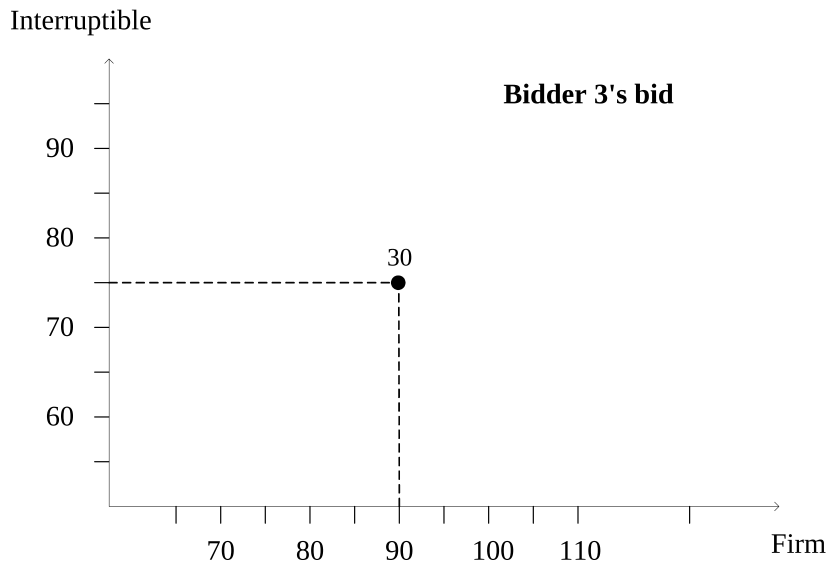
The type and quantity of contracts that will be allocated to bidder 1 depend on the final auction prices. For example, suppose that the auction price for firm contracts is 70 and the auction price for interruptible contracts is 65. Then both bids by bidder 1 are winning bids, since the prices in both bids are higher than the auction prices. Moreover, bidder 1 is allocated 30 units of firm contracts at price 70 and no interruptible contract, because firm contracts give bidder 1 a higher surplus. The reason is that, given his bid for 10 units of either firm or interruptible contracts and given the auction prices, bidder 1's per-unit surplus if he is allocated firm contracts is $80-70=10$, while his per-unit surplus if he is allocated interruptible contracts is $70-65=5$. This can be clearly seen in the following picture, where bidder 1's bid is to the south of the $45°$ line originating from the point that represents the auction prices for the two contracts.



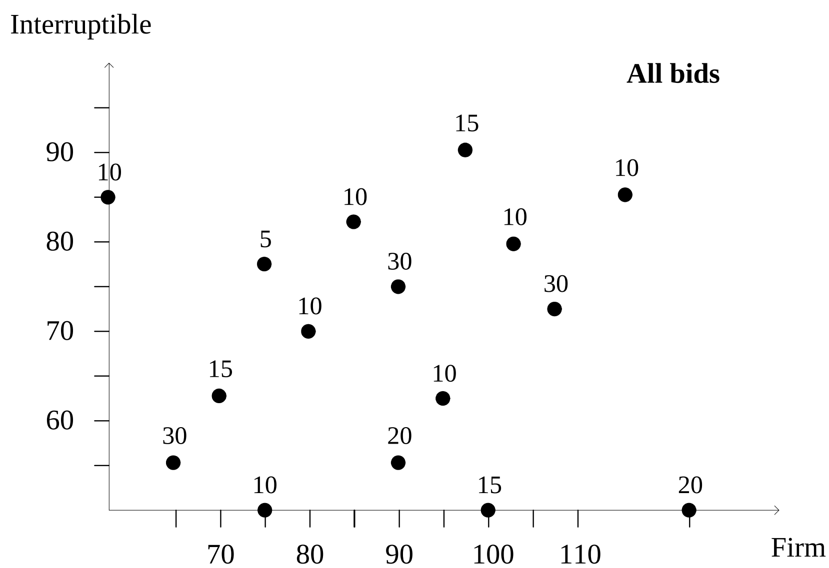
Similarly, bidder 2's and bidder 3's bids, as described in Section 4.2.1, are represented in the following two pictures. Bidder 2 makes 2 bids: one bid for 15 units of firm contracts at price 100, and one bid for 10 units of firm contracts at price 75.



Bidder 3 makes a single bid for either 30 units of firm contract at price $v_3^F = 90$, or 30 units of interruptible contracts at price $v_3^I = 75$.

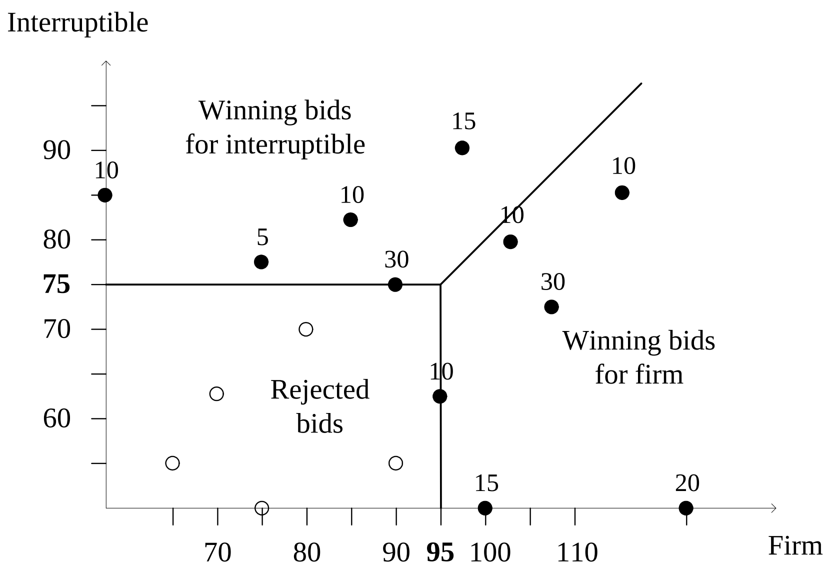


Consider now the auctioneer's choice of firm and interruptible prices. Assume all bids submitted by bidders in the auction are represented in the following picture.



The auctioneer has to choose the prices of firm and interruptible contracts so that the total quantity of firm contracts sold in the auction is equal to 90 (the supply of firm contracts) and the total quantity of interruptible contracts sold in the auction is equal to 60 (the supply of interruptible contracts). It is straightforward to check that the unique prices that achieve this objective are 95 for firm and 75 for interruptible contracts. Hence, these are the prices that will be paid by all winning bidders for each unit of the two types of contracts.

The winning bids are all those whose price are either higher than 95 for firm, or higher than 75 for interruptible, or both. Rejected bids are contained in a rectangle with one vertex at the origin of the axis and another vertex at the point identified by the auction prices for the two types of contracts. The auction prices and the allocations of firm and interruptible contracts are represented in the following picture.



Auction prices and allocation

Each bidder obtains the contract that gives him the highest margin, given his bids and the auction prices. Therefore, winning bids that are to the south of the 45° line originating from the point (95, 75) — that represents the auction prices for the two contracts — are allocated firm contracts (since the difference between the bid price and the auction price is higher for firm than for interruptible contracts). And winning bids that are to the north of the 45° line originating from the point (95, 75) are allocated interruptible contracts (since the difference between the bid price and the auction price is higher for interruptible than for firm contracts). Notice that: (i) in order to equalize the total demand to the total supply for interruptible contracts, the bidder who bid for 30 units of interruptible contracts at price 75 (that is equal to the auction price) is rationed and is only allocated 20 units of interruptible contracts; (ii) in order to equalize the total demand to the total supply for firm contracts, the bidder who bid for 10 units of firm contracts at price 95 (that is equal to the auction price) is rationed and is only allocated 5 units of interruptible contracts.

Klemperer (2010) provides additional details about the geometric representation of the product-mix auctions.