Addendum to "Reviewing the Financial Terms of Channel 3 Licences: Estimating Incumbents' Bids"

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1 Introduction

Ofcom has responded to our 1st August paper, "Reviewing the Financial Terms of Channel 3 Licences: Estimating Incumbents' Bids," with two questions concerning the equilibrium bidding behaviour of the incumbent in the Engelbrecht-Wiggans, Milgrom and Weber (1983) model, which we extended to allow for the costs of market entry.[1] The first question concerns the assumption, made in our examples, that the range of valuations of the entrants has a lower bound of zero, so is an interval of the form $[0, \overline{v}]$, where $\overline{v}$ is the highest value which entrants believed the ITV licences may be worth, and 0 the lowest value. Ofcom wishes to know how the incumbent's expected equilibrium bid would be altered if the range of beliefs were an interval of the form $[\underline{v}, \overline{v}]$ with $\underline{v} > 0$. Secondly, Ofcom has questioned the use of a uniform distribution over this range to represent entrants' beliefs in the particular examples we calculated.

We have no information which would allow us to predict what ITV would believe about a potential entrant if a Channel 3 licence auction were to be held. Estimating the range and the shape of the probability distribution over the possible valuations of a potential rival that ITV would employ in such an auction seems to us to present a major scientific problem. Who can say right now what ITV might or might not believe under hypothetical circumstances that have yet to be determined? Even if one could estimate ITV's beliefs, how would it be possible to guess at what an entrant might believe about ITV's beliefs? We agree that it is certainly possible that ITV's range of possible valuations might be bounded away from zero, and rather unlikely that its probability distribution would be uniform over its range. However, in offering a preliminary analysis of such examples, as suggested by Ofcom, we are anxious that it be understood that we are not thereby endorsing the principle that any particular probability distribution attributed to ITV in these examples would be common knowledge if an auction were to be held today.

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[1] Cases 6 and 7 in Section 5 of our 1st August paper.

2 Fundamental Issues

Before turning to the new examples in Section 3, it is worth reiterating and expanding on a number of issues that are more fundamental to the debate concerning the expected outcome of a hypothetical auction for Channel 3 licences, and which now risk being overlooked.

A. Off-the-Shelf Design? Our first point is that it was never our intention to propose a particular 'off-the-shelf' auction model as providing an adequate solution to Ofcom's problem of estimating the incumbents' bid in a hypothetical bidding process. For that, further detailed modelling work would be required, informed by reasonably credible estimates of the likely values of various parameters,[2] possibly followed by carefully designed simulations or experiments. The highly simplified examples discussed in our paper were intended to inform the discussion by illustrating the range of possible outcomes which can be derived from an appeal to auction theory alone. In particular, they served to cast considerable doubt on the solution proposed by Ofcom, which corresponded to the case of an auction with perfect information in which entry occurs with probability one. In our view, reviewing the relevant auction theory is simply a necessary first step in arriving at a credible estimate of the likely value of an incumbent's bid in the hypothetical licence auction being considered. It is especially useful for ruling out particularly unlikely predictions (such as those proposed in Ofcom's February and June consultation documents), but it would be reckless to rely on simple examples for any other purpose.

B. Combinatorial Auctions Our second point is that although the Engelbrecht-Wiggans, Milgrom and Weber (1983) model, as adapted in our paper, embodies a number of the essential features of a hypothetical Channel 3 licence auction,[3] the model is unrealistic in that it considers the sale of a single, indivisible item, and is not the combinatorial, multi-unit auction hypothesised by Ofcom. In our paper we pointed out that our use of such a model represented a simplifying assumption, which was made solely for the purposes of allowing us to explore some of the implications of Ofcom's proposed combinatorial auction within a reasonably tractable framework This solution fell far short of offering a realistic analysis of the actual combinatorial auction being proposed, however. Thus we wrote in Section 3:

"The theory of optimal bidding in combinatorial auctions is still in its infancy, and has only been worked out for the case of perfect information. ... Thus predicting equilibrium outcomes and bids in a fully-fledged combinatorial licence auction is probably beyond the limits of current auction theory. Even if we assume perfect information (which we must to apply the developed theory), the equilibrium outcomes may be inef-

[2] Such as, for example, the likely beliefs of the potential entrants as represented by the range $[\underline{v}, \overline{v}]$, and the relevant probability distributions describing these beliefs.

[3] In particular the incumbent's cost and informational advantages, as well as the fact that it is probably best represented as an auction with a large common values component.

ficient incumbent bidder loses, despite having the highest valuation) or may allow the incumbent to obtain the licences for a price well below his valuation." [4]

Since Ofcom is now suggesting that it is reasonable to assume that it would be able to hold a true combinatorial auction (in which the licences are awarded to the bids which maximise the auctioneer's overall revenue, rather than to the highest bidder for each individual licence),[5] it has become particularly important to emphasise that the relevant equilibrium theory for such an auction is practically nonexistent, making it nearly impossible to arrive at robust predictions of the likely outcomes without further detailed modelling work. Given that what theory exists on combinatorial, first-price auctions sometimes predicts inefficient outcomes with low prices, the results reported in Section 6 of our paper, and further extended here, can probably be considered as representing the very best that such a combinatorial auction could achieve, rather than as providing predictions of the likely auction outcomes.

There are further difficulties associated with combinatorial auctions which did not receive detailed treatment in our paper. The first is the "winner determination problem", which as Milgrom (2204) has pointed out, 'makes it hard for bidders in a combinatorial auction to forecast the consequences of their bids.' Ausubel and Milgrom (2002) expand on this point as follows:

> *"To understand the technical challenge, suppose that bidders submit bids for overlapping packages. Given these bids, the first step of finding the sets of "consistent" bids in which each individual item is included in just one package ("sold just once") is a hard computational problem. Then, the total bid associated with each such package must be computed and the revenue-maximizing set of "consistent" bids must be found."*

A second difficulty is the "valuation determination problem". If eleven Channel 3 licenses were offered for sale in a single combinatorial auction, each potential bidder would in principle need to consider his valuation for each of 2047 distinct combinations involving between one and eleven licenses. This is not the impractically huge number that would arise in some combinatorial auctions that have been proposed for use in the American telecoms industry, but it remains a very large number from the point of view of a potential entrant who must evaluate each licence separately, since they all have different characteristics. Ausubel and Milgrom (2002) also discuss the crucial importance of this issue:

[4] Hence, in the introduction to Section 5 on combinatorial auctions for multiple licences, we included the caveat: *"As noted in Section 3, full-fledged combinatorial auctions are very complex and extremely difficult to analyse, so for the purposes of discussing the hypothetical auction being contemplated we will simplify some of the issues. If we assume that each extra licence acquired increases the valuation for any licence yet to be auctioned, we would expect the winning assemblage of bids in a combinatorial auction to consist of a single conditional bid, in which the winner offers a cash sum for all of the licences on offer. In other words, we can treat the auction as if it were a first-price, sealed-bid auction of a single object: i.e. the package of all eleven licences treated as a unit. The remainder of this section assumes that this is the case, even though such an approach may be in conflict with the relevant legislation."*

[5] See Section 3 of our 1 August paper for the importance of this distinction.

"Another potentially significant issue is the cost of determining valuations. A traditional assumption in auction theory analyses is that each bidder knows all its values or can compute them at a zero cost. For package auctions, the sheer number of combinations that a bidder must evaluate makes that assumption especially dubious. Compared to most of the other costs involved in conducting combinatorial auctions, bidder valuation costs are relatively less affected by advancing technologies, particularly when the asset valuation process requires substantial human inputs. Potential buyers who find it too expensive to investigate every packaging alternative will instead choose a few packages to evaluate fully. Ideally, auction design should account for the way those choices are made as well as the evaluation costs that bidders incur. When package evaluation is costly, the interacting choices that bidders make about which packages to evaluate further complicate the analysis. ... In a Vickrey auction or any sealed bid auction, if it is too costly to evaluate all the packages, then bidders must guess about which packages are most relevant and how to allocate their limited evaluation resources."

Ausubel and Milgrom (2002) then go on to discuss the practical difficulties involved in such valuation exercises. Everything they say in the following passage would appear to apply equally to the valuation of ITV licences:

"In our experience, valuing significant business assets involves both investigating the asset itself and creating business plans showing how they will be used. For example, a bidder hoping to purchase parts of an electrical generating portfolio might investigate the physical condition of each plant, the availability of land and water for cooling to allow plant expansion, actual and potential transmission capacity, and other physical variables. In addition, it will consider labor and contractual constraints, zoning and other regulatory constraints, the condition of markets in which power might be sold, partnerships that might enhance the asset value, and so on. The final valuation is the result of an optimization over business plans using all this information, and tempered by human judgment. When the assets in the collection interact in complex ways that affect the optimal business plan, then significant extra costs must be incurred to evaluate each package."

The relevance to the current discussion is that running a combinatorial auction would add very significantly to the entry costs of a potential entrant, thereby worsening an entry problem whose seriousness we have emphasized in previous papers. Why should any challenger enter at all if the chances of winning a licence are small as a result of the substantial incumbent advantages enjoyed by the current holder of the licences? And the larger the entry costs incurred, the less attractive entry will be to any potential challenger.

A third difficulty is that even in the case of perfect information, where the equilibria can be derived, even very simple combinatorial auctions can have numerous equilibria, many of which may be inefficient or involve low revenues.[6] The following example illustrates the types of equilibria which can arise in a combinatorial auction with two bidders, two items for sale and perfect information. Table 1 shows the valuations of the two bidders, I (the "incumbent") and E (the "entrant") for 2 licences, A and B. Common ownership is assumed to increase the value of winning both licences by 1 for each bidder.

Table 1

Licence Values	Licence A	Licence B	A and B
Bidder I	5	1	7
Bidder E	0	4	5

Table 2 shows one equilibrium for the combinatorial auction in which Licence A is awarded to Bidder I and Licence B to Bidder E (on the assumption that ties are resolved efficiently). This allocation is efficient, however bidder I's individual and conditional (i.e. package) bids are both significantly less than half of his respective valuations. The auctioneer receives a total revenue of 3 in this equilibrium, while the bidders' joint profits are 6, so the money 'left on the table' by the auction is 6 out of a total value of 9.

Table 2

Equilibrium Bids	Licence A	Licence B	A and B
Bidder I	1	0	3
Bidder E	0	2	3

Table 3 illustrates another equilibrium in which both licences are awarded to Bidder I for a total payment of 5 (again on the assumption that ties are resolved efficiently). This equilibrium increases the auctioneer's revenues, but results in an inefficient allocation.

Table 3

Equilibrium Bids	Licence A	Licence B	A and B
Bidder I	0	0	5
Bidder E	0	0	5

Many other equilibria exist for this example (see Bernheim and Whinston, 1986), making prediction difficult. And the problems posed by such simple combinatorial auctions increase exponentially as the number of items for sale increases.

C. The Entry Problem Finally, we return to the entry problem mentioned above. Recall that the prediction of most of the models we considered in which the incumbent bidder had either a cost or informational advantage (or both), was that entry into the auction would not occur in the presence of any small auction entry cost.[7] This is a robust prediction for auctions of the type considered in Examples 6 and 7 of our paper, as demonstrated by Theorem 2 of Engelbrecht-Wiggans, Milgrom and Weber (1983). This theorem encompasses affiliated in addition to pure common values, information asymmetries between the entrants, and differing degrees of risk aversion. It demonstrates that under a wide variety of conditions the expected equilibrium payoff

[6] This was first demonstrated by Bernheim and Whinston (1986), upon whose analysis the following example has been based.

[7] Auction entry costs need to be carefully distinguished from market entry costs.

of a disadvantaged or less well-informed bidder in a first-price, sealed-bid auction is zero. Hence for any positive cost of entering the auction, entry will not occur, and the incumbent bidder will win the auction for a nominal amount.

It is difficult to overemphasise the importance of this result for the current discussion, as it means that the prediction of many of the models which incorporate the types of asymmetries we would expect to observe in a hypothetical Channel 3 licence auction is that the incumbent bidder will face no competition at all. As Ofcom itself has noted (June consultation document, para 27), the incumbent's bid in a hypothetical licence auction will depend upon the degree of competition it expects to face. Since in at least 3 of the 16 Channel 3 licence auctions held in 1991 incumbent bidders faced no competition at all, this prediction of auction theory cannot simply be brushed aside in favour of an outcome which is more to Ofcom's liking.

3 Further Examples of Equilibrium Bidding in a First-Price Auction with Market Entry Costs

This section offers a preliminary analysis of the model of Engelbrecht-Wiggins, Milgrom and Weber (1983), modified as suggested by Ofcom. We consider both the case when the range of the challenger's valuations is bounded away from zero, and cases when its probability distribution over valuations is not uniform. Our reservations about making an essentially arbitrary choice of such an off-the-shelf model remain valid. (We have good reason, for example, to doubt that a hypothetical auction of Channel 3 licences could be realistically regarded as a pure common-value auction).

The model we consider concerns a common-value, first-price auction in which one bidder (referred to as *the incumbent)*, knows the value of the object for sale is V_I, and n other bidders (*the entrants)* who possess imperfect information about this value. Any potential entrant who wins the object also incurs a fixed market entry cost $c \geq 0$, which is assumed to be the same for all entrants. The entrants' information about the incumbent's valuation is described by a distribution F, with density f, on support $[\underline{v}, \overline{v}]$. To analyse this situation we must further assume that the market entry costs c and the entrants' beliefs F are common knowledge. To simplify the analysis we shall also assume that $\underline{v} > c$. This means that even in the worst case the entrants believe that the object has positive value.[8]

A pure strategy for the incumbent in the auction is a bid function b_I which specifies the incumbent's bid for each possible valuation in $[\underline{v}, \overline{v}]$. A pure strategy for an entrant is simply a number $b_E \geq 0$. A mixed strategy for an entrant is a distribution function specifying the probabilities with which the entrant chooses each of the possible bids.

[8] To characterize the equilibrium of the auction, we must also specify a *tie-breaking rule* which determines how the object is allocated if more than one bidder submits the highest bid. We shall adopt the natural assumption that in this situation the object is allocated with equal probability to any of the bidders who submitted the highest bid.

3.1 The Equilibrium of the Auction

In this section we derive an equilibrium for the auction, which we then use in Section 4 to calculate some numerical results based on specific examples. The following strategies constitute an equilibrium of the game:

- Incumbent bids, $b_I(v) = E[V_I|V_I \leq v] - c$.
- Each entrant uses the same mixed strategy characterized by a distribution function H with support $[\underline{v} - c, E[V_I] - c]$, where for any bid b in the support:

$$H(b) \equiv \sqrt[n]{e^{-\int_b^{E[V_I]-c} \frac{1}{\phi(\tilde{b})-\tilde{b}} d\tilde{b}}},$$

  where $\phi(b)$ is equal to the inverse of $b(v)$, i.e. $E[V_I|V_I \leq \phi(b)] - c = b$.

To demonstrate that the above strategies are indeed an equilibrium we first show that the entrants have no incentive to deviate. To see this, note first that no entrant would wish to bid above the maximum bid of the incumbent $E[V_I]-c$. Doing so implies that they do not learn anything about the value of the object, and hence their expected value will be equal to $E[V_I] - c$. Further, any bid less than or equal to $E[V_I] - c$ earns the entrant zero expected utility. A bid below $\underline{v} - c$ implies losing the auction with probability 1. Any bid $b \in [\underline{v} - c, E[V_I] - c]$ also gives zero expected utility since if an entrant wins by bidding b,her expected utility is equal to $E[V_I|b_I(V_I) \leq b] - c - b$. If we denote by v the valuation which yields $b_I(v) = b$,then this expected utility can be rewritten as $E[V_I|V_I \leq v] - c - b_I(v)$. This is equal to zero by definition of the incumbent's bid function. Consequently, the entrants are indifferent between all bids less than or equal to $E[V_I] - c$ and have no incentive to deviate from the specified equilibrium strategy.

Next we must show that given the distribution of the entrants' bids, $H(b)$, an incumbent with value $v \in [\underline{v}, \overline{v}]$ finds it optimal to bid $b_I(v)$. Clearly the incumbent has no incentive to bid above $E[V_I] - c$ nor to bid below $\underline{v} - c$.[9] Consequently the incumbent's optimal bid must lie in the interval $[\underline{v} - c, E[V_I] - c]$. The incumbent's expected utility from a bid b in this interval is equal to $(v - b)H(b)^n$ whenever $H(b)$ has no atoms at b.[10]

Consider first the case in which $c = 0$. Then H has no atoms and the incumbent's optimal bid can be computed from the first-order conditions. Simple algebra shows that the optimal bid is indeed $b_I(v)$.[11] If $c > 0$ then $H(b)$ has an atom at $b = \underline{v} - c$ and the incumbent's objective function is not continuous at $b = \underline{v} - c$. We can carry out the same analysis as in the preceding case ($c = 0$) if we can show that the bid $\underline{v} - c$ is always suboptimal. To see this, note that the fact that $H(p)$ has an atom of

[9] The former is because such bids have the same probability of winning as a bid $E[V_I]-c$, but imply paying a higher price, and the latter because such bids imply losing the auction with probability one.

[10] The distribution H may have atoms only at $b = \underline{v} - c$. In that case the uniform tie-breaking rule the yields the incumbent's expected payoff is $(v - b)\frac{H(b)^n}{n}$.

[11] The first-order conditions are sufficient since the incumbent's expected payoffs are supermodular in his bid and his valuation. Supermodularity means that the cross-derivative with respect to b and v is positive, i.e. that incumbents with higher values have larger incentives to bid higher.

probability at $\underline{v} - c$ means that there is a positive probability that every entrant will bid $\underline{v} - c$. However, because of our tie-breaking rule, the incumbent can always get higher expected utility with a bid slightly higher than $\underline{v} - c$ than by bidding $\underline{v} - c$. Consequently, $b_I(v)$ is the incumbent's optimal bid function when the entrants bid according to H.

3.2 Computations

3.2.1 Analysis of the Incumbent's Bid Function

We now compute the incumbent's equilibrium bid for a number of different distribution functions

Uniform Distribution We start with the simplest, and probably the most reasonable, distribution function, the uniform. In the uniform case entrants attach equal probability to any v in the range $[\underline{v}, \overline{v}]$. It is straightforward to calculate that:

$$b(v) = \frac{V_I + \underline{v}}{2} - c$$

and,

$$H(b) = \sqrt[n]{\frac{b + 2c - \underline{v}}{\frac{\overline{v} - \underline{v}}{2} + c}}$$

for any b in the support $[\underline{v} - c, \frac{\underline{v} + \overline{v}}{2} - c]$.

The Beta Case We next consider a distribution function which is symmetric around the mean and has either an inverted "U"-shaped density or a "U"-shaped density function. In the first case, entrants believe the incumbent's valuation to be concentrated around the mean, whereas in the second case entrants believe that the incumbent's valuation is either very high or very low, but place little weight on intermediate values.

A beta distribution function is characterized by two parameters, and it is symmetric whenever the two parameters take the same value. If this value exceeds one, the distribution has an inverted "U" shape, and if it is less than one, a "U" shape. For the case, in which both parameters are equal to one, the beta distribution is actually a uniform distribution.

Numerous simulations suggest that whenever the parameters of the beta are above 1, i.e. an inverted "U" shape, the incumbent's bid function lies slightly above the bid function associated with the uniform distribution, whereas when the parameters are below 1, i.e. an "U" shape, the incumbent's bid function lies slightly below the uniform distribution bid function.

The Extreme-Linear Case Finally, we consider two other cases in which entrants' beliefs concentrate probability on high or low values in $[\underline{v}, \overline{v}]$. We consider two cases in which the density is linear. The first case is a density $f(v) = 2\frac{\overline{v} - v}{(\overline{v} - \underline{v})^2}$ and the second

$f(v) = 2\frac{v-\underline{v}}{(\overline{v}-\underline{v})^2}$. We shall call the first case, "extreme linear bias towards the upper bound", and the second "extreme linear bias towards the lower bound".

The incumbent's bid function corresponding to the "extreme linear bias towards the upper bound" is equal to $b_I(v) = \frac{2v-\underline{v}}{\overline{v}-\underline{v}} - c$. This bid function lies above the incumbent's bid function corresponding to the uniform distribution. The bid function which corresponds to the "extreme linear bias towards the lower bound" is more complicated, and equal to,

$$b_I(v) = \frac{v(3\overline{v} - 2v) + \underline{v}(3\overline{v} - 2(\underline{v} + v))}{3(2\overline{v} - (v + \underline{v}))} - c.$$

This bid function lies below the uniform distribution case.

3.2.2 Analysis of the Incumbent's Expected Bid

In the uniform case, it is quite simple to compute the expected bid of the incumbent, since the bid function is linear. In particular, it is equal to the bid corresponding to $v = E(V_I)$, i.e. $\frac{\overline{v}+3\underline{v}}{4} - c$.

In the other cases, it is more difficult to obtain closed form solutions. To get an idea of the range of possible outcomes we provide some computations. The incumbent's expected bid for each distribution function is given by:

- For the uniform distribution: $E(b_I(V_I)) = 0.25(\overline{v} - \underline{v}) + \underline{v} - c$.
- For the Beta distribution with parameter 1.25: $E(b_I(V_I)) = 0.26(\overline{v} - \underline{v}) + \underline{v} - c$.
- For the Beta distribution with parameter 0.75: $E(b_I(V_I)) = 0.23(\overline{v} - \underline{v}) + \underline{v} - c$.
- For the extreme linear bias towards the upper bound: $E(b_I(V_I)) = 0.33(\overline{v} - \underline{v}) + \underline{v} - c$.
- For the extreme linear bias towards the lower bound: $E(b_I(V_I)) = 0.20(\overline{v} - \underline{v}) + \underline{v} - c$.

3.3 Conclusion

The analysis of further examples of the Engelbrecht-Wiggans, Milgrom and Weber model has demonstrated that the exact estimate of the incumbent's bid will depend upon what we assume about the entrants' beliefs, the incumbent's beliefs about these beliefs, and so on. None of this justifies the solution proposed by Ofcom to estimating the incumbent's bid in a hypothetical licence auction. For example, if $[\underline{v}, \overline{v}] = [10, 20]$, then our highest estimate of the incumbent's bid is 11.8 and our lowest estimate 10.5 (assuming $c = 1.5$). Taking $[\underline{v}, \overline{v}] = [10, 120]$ yields a highest estimate of 39.8 and a lowest estimate of 25.5 (assuming $c = 6.5$). Ofcom's estimates for the uniform distribution in these cases would be 13.5 and 58.5 respectively.

There is little point, however, in carrying on with these calculations in the absence of any reliable information on the relevant parameters and probability distributions, and without attempting a more serious modelling exercise to capture essential features of Ofcom's proposed combinatorial auction which the current model assumes away. As we stated above, an appeal to auction theory and simple examples is useful for eliminating untenable predictions, but it falls far short of providing an adequate solution to Ofcom's problem of estimating ITV's' bid in a hypothetical auction for Channel 3 licences.

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